QuantGANs & DDPG

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Notation

Adjusted closing price 조정 종가

The closing price of i^{th} asset after period t is $\boldsymbol{v_{i,t}^{close}}$.

종가

The closing price of all assets comprise the price vector for period t as $\boldsymbol{v_t^{close}}$.

Notation

$\frac{v_t}{v_{t-1}}$

$$y_t = \frac{\mathbf{v}_t}{\mathbf{v}_{t-1}} = \left(1, \frac{v_{1,t}}{v_{1,t-1}}, \cdots, \frac{v_{m,t}}{v_{m,t-1}}\right)^T$$ is the price fluctuating vector

w_{t-1}

$\mathbf{w}_{t-1} = (w_{0,t-1}, w_{1,t-1}, \cdots, w_{m,t-1})^T$ represents the reallocated weight at the end of time $t-1$ with constraint $\sum_i w_{i,t-1} = 1$

실제 배분 비율 (자산 보유 비율)

a_{t-1}

$a_{t-1} = (a_{0,t-1}, a_{1,t-1}, \cdots, a_{m,t-1})^T$ represents the desired allocating weight at the end of time $t-1$ with constraint $\sum_{i=0}^{n} a_{i,t-1} = 1$

목표 배분 비율

- Reward(r): the naive fluctuation of wealth minus transaction cost. The fluctuation of wealth is $a_{t-1}^T \cdot y_{t-1}$. In the meanwhile, transaction cost should be subtracted from that, which equals $\mu \sum_{i=1}^{m} |a_{i,t-1} - w_{i,t-1}|$. The equation above suggests that only transactions in stocks occur transaction cost. Specifically, we set $\mu = 0.25\%$. In conclusion, the immediate reward at time t-1 as:

$$r_t(s_{t-1}, a_{t-1}) = \log(\underbrace{\mathbf{a_{t-1} \cdot y_{t-1}}}_{\text{fluctuation of wealth}} - \underbrace{\mu \sum_{i=1}^{m} |a_{i,t-1} - w_{i,t-1}|}_{\text{transaction cost}}) \quad \textbf{+ epsilon (=10e-8)}$$

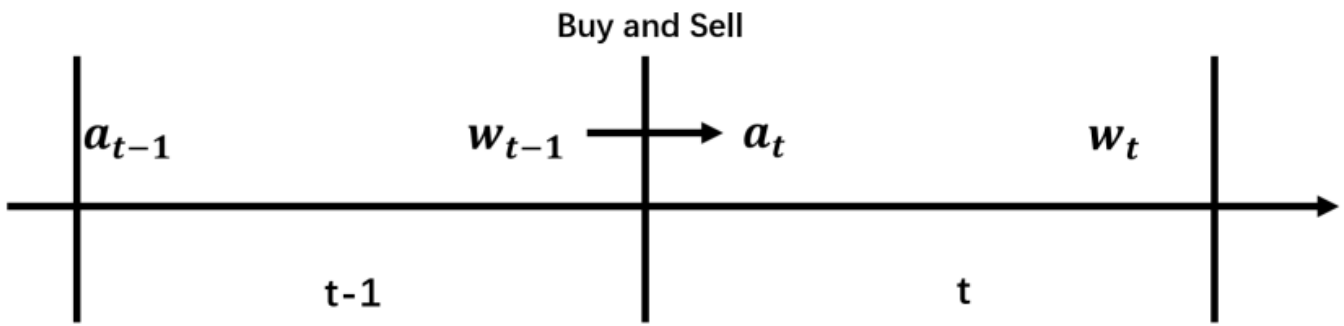
- State(s): one state includes previous open, closing, high, low price, volume or some other financial indexes in a fixed window.

Adjusted close

```
[**********100%%**********]  1 of 1 completed
                 Open         High          Low        Close    Adj Close  \
Date
2009-01-02   902.989990   934.729980   899.349976   931.799988   931.799988
2009-01-05   929.169983   936.630005   919.530029   927.450012   927.450012
2009-01-06   931.169983   943.849976   927.280029   934.700012   934.700012
2009-01-07   927.450012   927.450012   902.369995   906.650024   906.650024
2009-01-08   905.729980   910.000000   896.809998   909.729980   909.729980
...                 ...          ...          ...          ...          ...
2019-12-23  3226.050049  3227.780029  3222.300049  3224.010010  3224.010010
2019-12-24  3225.449951  3226.429932  3220.510010  3223.379883  3223.379883
2019-12-26  3227.199951  3240.080078  3227.199951  3239.909912  3239.909912
2019-12-27  3247.229980  3247.929932  3234.370117  3240.020020  3240.020020
2019-12-30  3240.090088  3240.919922  3216.570068  3221.290039  3221.290039

               Volume
Date
2009-01-02  4048270000
2009-01-05  5413910000
2009-01-06  5392620000
2009-01-07  4704940000
2009-01-08  4991550000
...                ...
2019-12-23  3064530000
2019-12-24  1296530000
2019-12-26  2164540000
2019-12-27  2429150000
2019-12-30  3021720000

[2767 rows x 6 columns]
```

- Action(a): the desired allocating weights, $a_{t-1} = (a_{0,t-1}, a_{1,t-1}, \ldots, a_{m,t-1})^T$ is the allocating vector at period $t-1$, subject to the constraint $\sum_{i=0}^{n} a_{i,t-1} = 1$. Due to the price movement in a day, the weights vector a_{t-1} at the beginning of the day would evolve into w_{t-1} at the end of the day:

$$\mathbf{w_{t-1}} = \frac{\mathbf{y_{t-1}} \odot \mathbf{a_{t-1}}}{\mathbf{y_{t-1}} \cdot \mathbf{a_{t-1}}}$$



Ornstein-Uhlenbeck Process

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck process x_t is defined by the following stochastic differential equation:

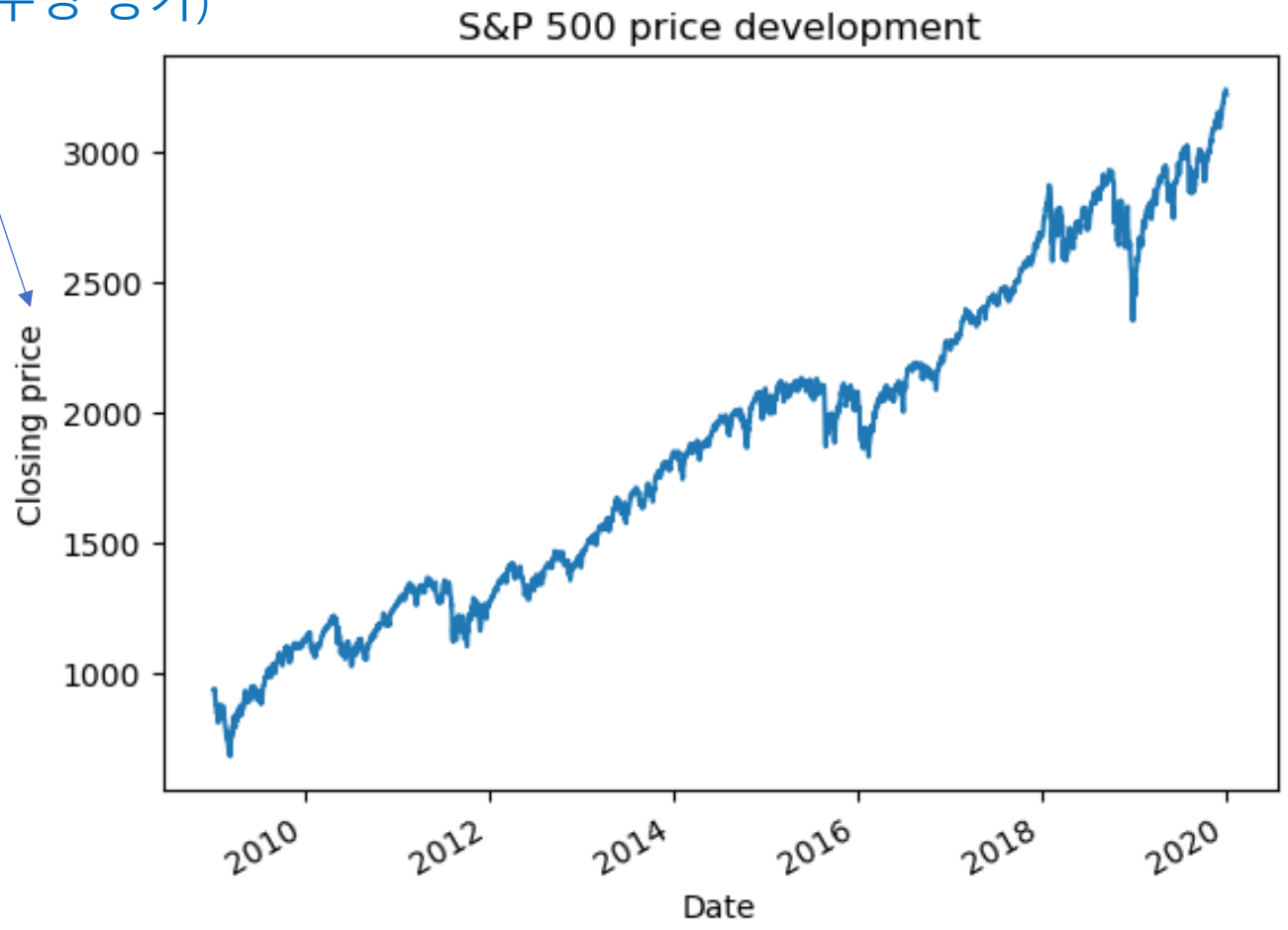
$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t$$

where $\theta > 0$ and $\sigma > 0$ are parameters, μ is drift term and W_t denotes the Brownian motion.

μ=0.0, θ=0.15, σ=0.2, dt=1e-2

N: number of features **(N=1)**

Adjusted close (수정 종가)



M, L


3 generated log return paths
Cumalative log return
0.4
0.2
0.0
−0.2
−0.4
0
500
1000
1500
2000
2500
days

Actor

```
class DDPGActor(nn.Module):
    def __init__(self, M, L, N): # pass number of assets (M) for defining the network
        super(DDPGActor, self).__init__()
        self.M = M
        self.L = L
        self.N = N
        self.basic = cnnNetwork(self.M, self.L, self.N)
        self.fc1 = nn.Linear(self.M, self.M)
        self.softmax = nn.Softmax()

    def forward(self, input):
        x, w = input
        # x = x.permute(0, 3, 1, 2)
        y = self.basic(x)
        y = self.fc1(torch.add(y,w))
        y = self.softmax(y)
        return y
```

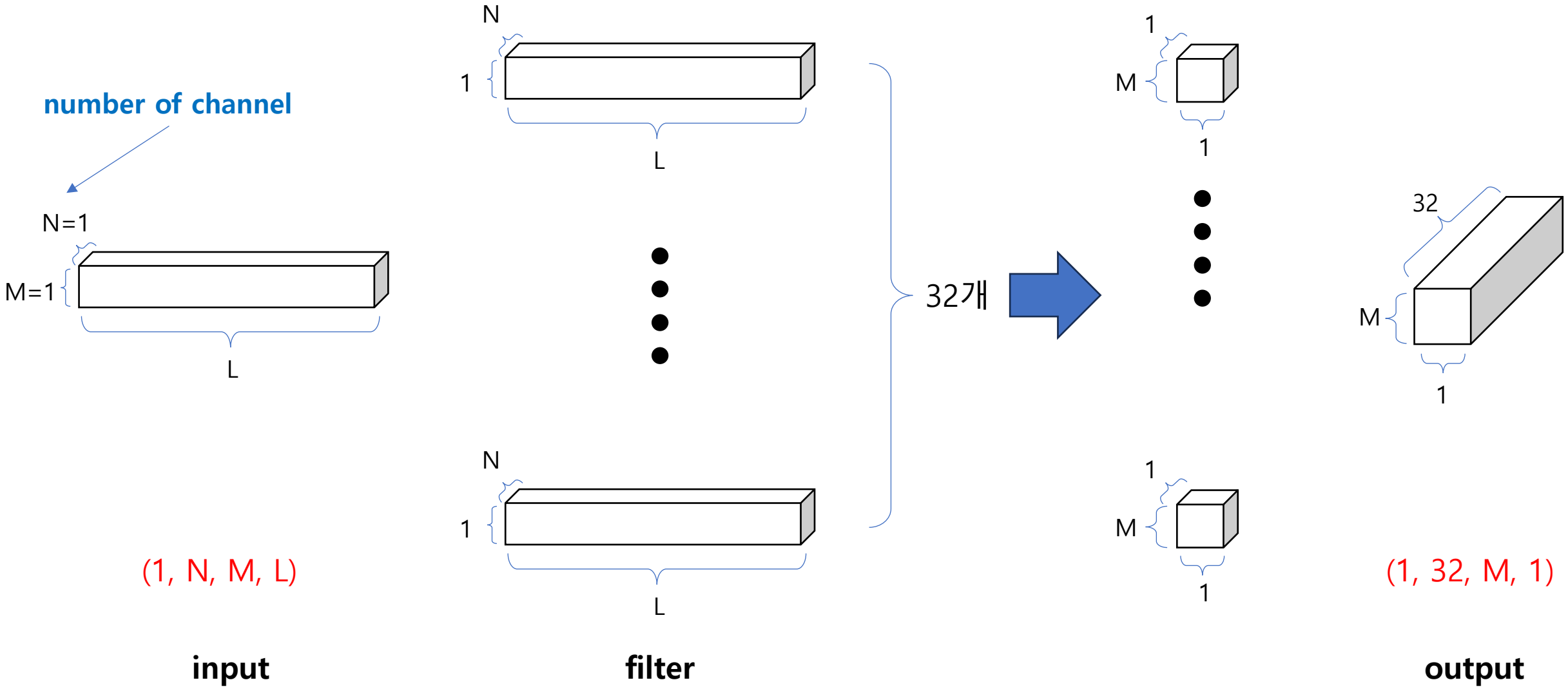
x: state, w: action

g(f (state) + action)

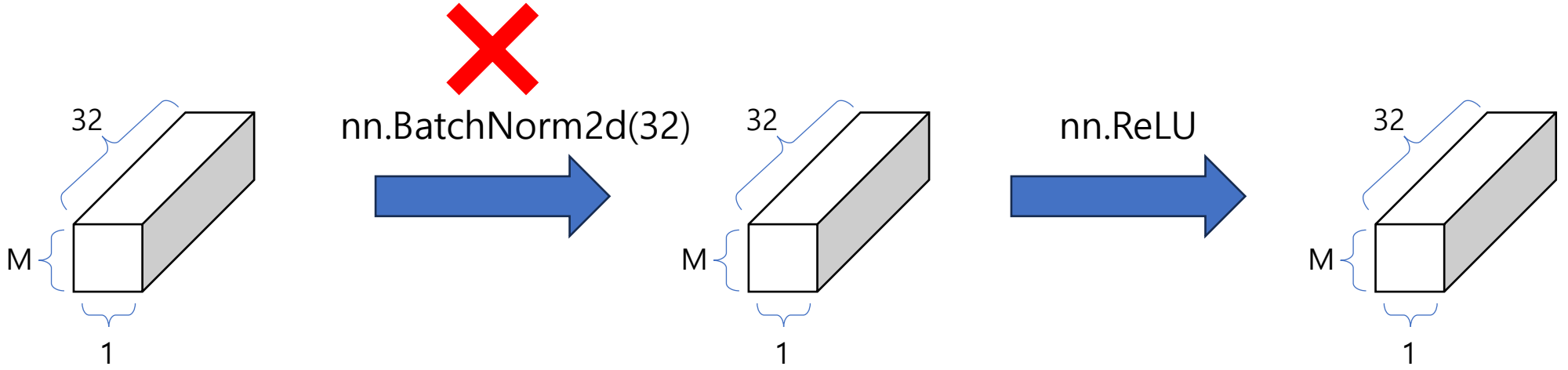
cnnNetwork

```
class cnnNetwork(nn.Module):
    def __init__(self, M, L, N):
        super(cnnNetwork, self).__init__()
        self.M = M
        self.L = L
        self.N = N

        self.conv2d1 = nn.Conv2d(in_channels=self.N, out_channels=32,
                                 kernel_size = (1,self.L), stride = 1, padding = "valid", bias=False)
        self.batchnorm1 = nn.BatchNorm2d(32)
        self.relu = nn.ReLU()

        self.conv2d2 = nn.Conv2d(in_channels=32, out_channels=1,
                                 kernel_size = (1,1), stride = 1, padding = "valid", bias=False)
        self.batchnorm2 = nn.BatchNorm2d(1)
        self.flatten = nn.Flatten()

    def forward(self, x):
        y = self.conv2d1(x)
        y = self.relu(self.batchnorm1(y))
        y = self.conv2d2(y)
        y = self.relu(self.batchnorm2(y))
        y = self.flatten(y)
        return y
```

```
self.conv2d1 = nn.Conv2d(in_channels=self.N, out_channels=32, kernel_size = (1,self.L), stride = 1, padding = "valid", bias=False)
```



```
self.batchnorm1 = nn.BatchNorm2d(32)
self.relu = nn.ReLU()
```

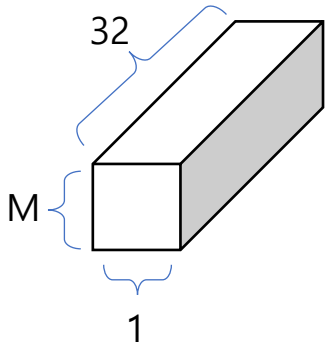


(1, 32, M, 1)

(1, 32, M, 1)

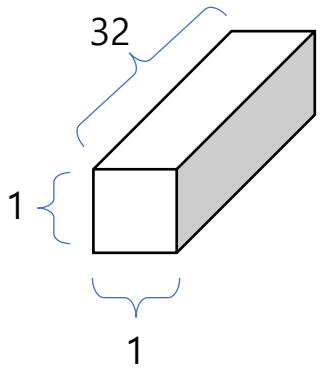
(1, 32, M, 1)

```
self.conv2d2 = nn.Conv2d(in_channels=32, out_channels=1, kernel_size = (1,1), stride = 1, padding = "valid", bias=False)
```

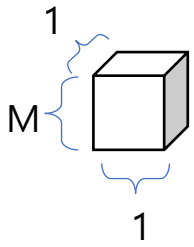


(1, 32, M, 1)

input



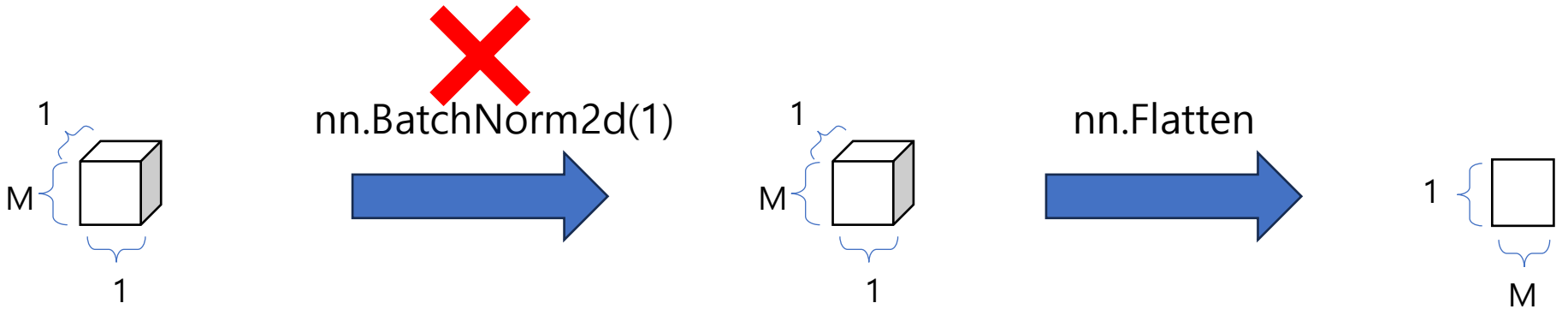
filter



(1, 1, M, 1)

output

```
self.batchnorm2 = nn.BatchNorm2d(1)
self.flatten = nn.Flatten()
```



(1, 1, M, 1)

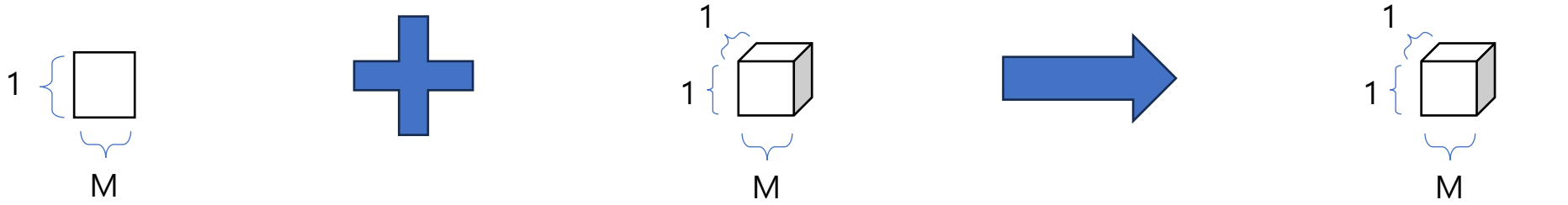
(1, 1, M, 1)

(1, M)

Actor

```
class DDPGActor(nn.Module):
    def __init__(self, M, L, N): # pass number of assets (M) for defining the network
        super(DDPGActor, self).__init__()
        self.M = M
        self.L = L
        self.N = N
        self.basic = cnnNetwork(self.M, self.L, self.N)
        self.fc1 = nn.Linear(self.M, self.M)
        self.softmax = nn.Softmax()

    def forward(self, input):
        x, w = input
        # x = x.permute(0, 3, 1, 2)
        y = self.basic(x)
        y = self.fc1(torch.add(y,w))
        y = self.softmax(y)
        return y
```

torch.add(y, w)



(1, M)

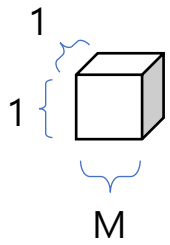
(1, 1, 1 ,M)

(1, 1, 1 ,M)

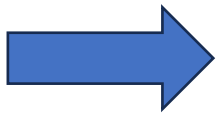
action (w)

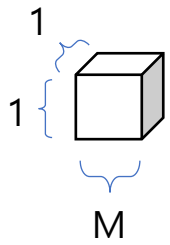
```
self.fc1 = nn.Linear(self.M, self.M)
```

```
y = self.fc1(torch.add(y,w))
```



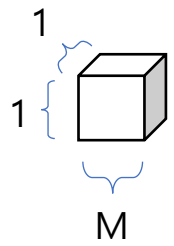
(1, 1, 1 ,M)





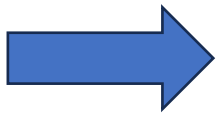
(1, 1, 1 ,M)

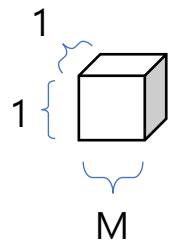
```
y = self.softmax(y)
```



(1, 1, 1 ,M)

softmax





(1, 1, 1 ,M)

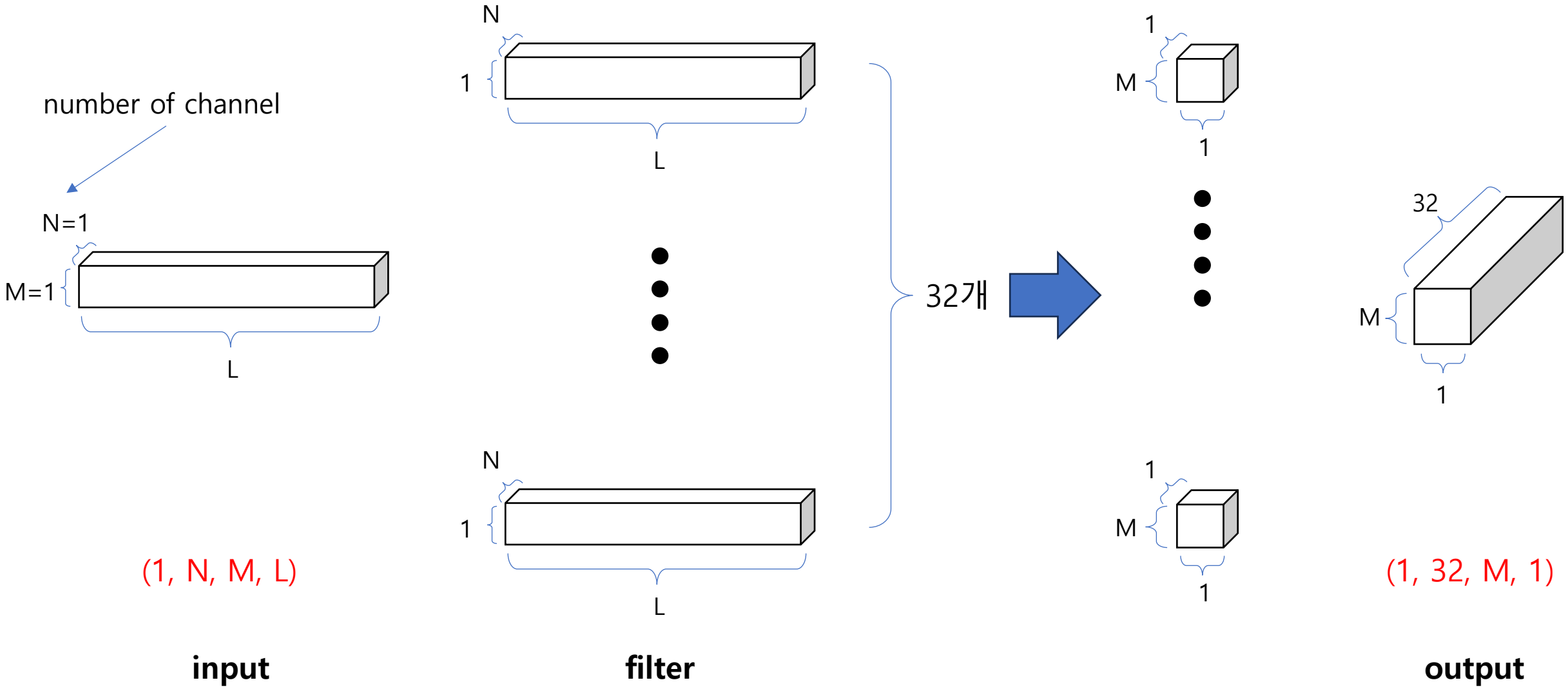
Critic

```
class DDPGCritic(nn.Module):
    def __init__(self, M, L, N):
        super(DDPGCritic, self).__init__()
        self.M = M
        self.L = L
        self.N = N
        self.basic = cnnNetwork(self.M, self.L, self.N)
        self.fc1 = nn.Linear(self.M, 1)

    def forward(self, input):
        x, w , action = input
        # x = x.permute(0, 3, 1, 2)
        y = self.basic(x)
        y = torch.add(y, action)
        y = torch.add(y, w)
        y = self.fc1(y)
        return y
```

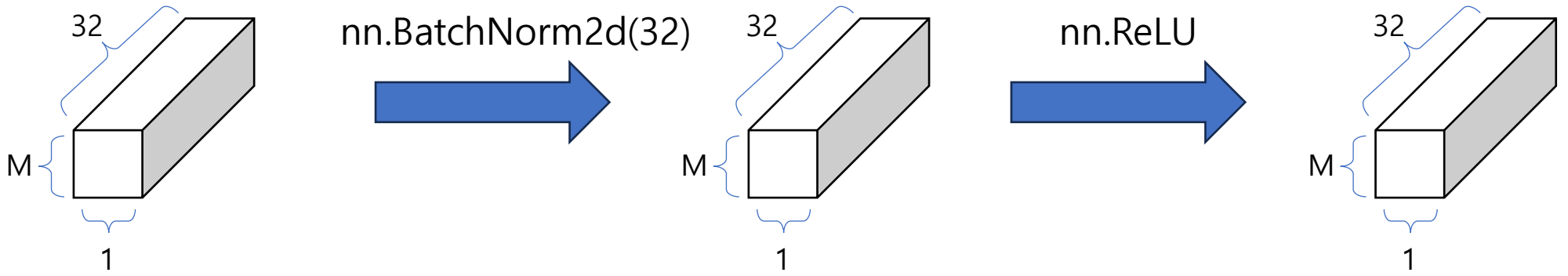
x: state, w: previous_action, action: action

g(f(state) + action + previous_action)

```
self.conv2d1 = nn.Conv2d(in_channels=self.N, out_channels=32, kernel_size = (1,self.L), stride = 1, padding = "valid", bias=False)
```



```
self.batchnorm1 = nn.BatchNorm2d(32)
self.relu = nn.ReLU()
```

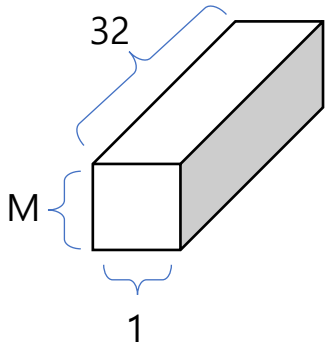


(1, 32, M, 1)

(1, 32, M, 1)

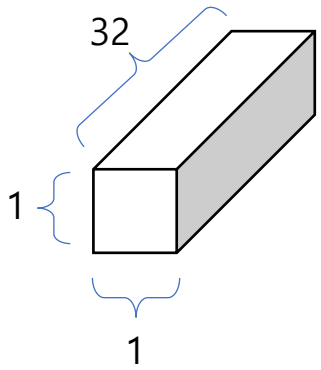
(1, 32, M, 1)

```
self.conv2d2 = nn.Conv2d(in_channels=32, out_channels=1, kernel_size = (1,1), stride = 1, padding = "valid", bias=False)
```

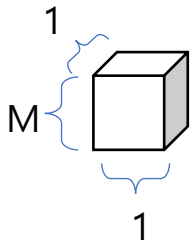


(1, 32, M, 1)

input



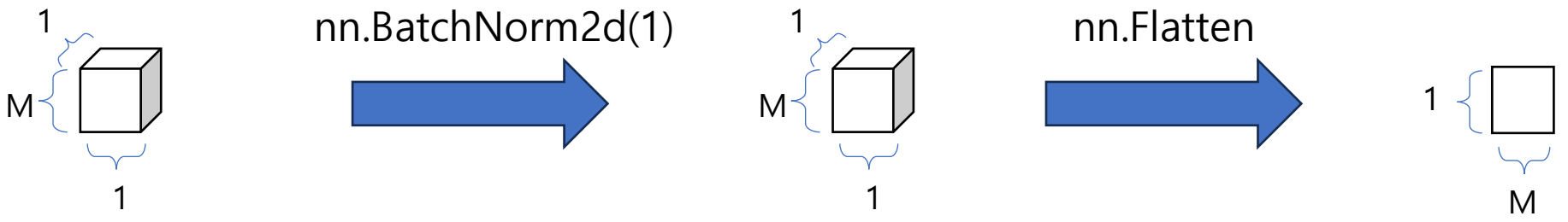
filter



(1, 1, M, 1)

output

```
self.batchnorm2 = nn.BatchNorm2d(1)
self.flatten = nn.Flatten()
```

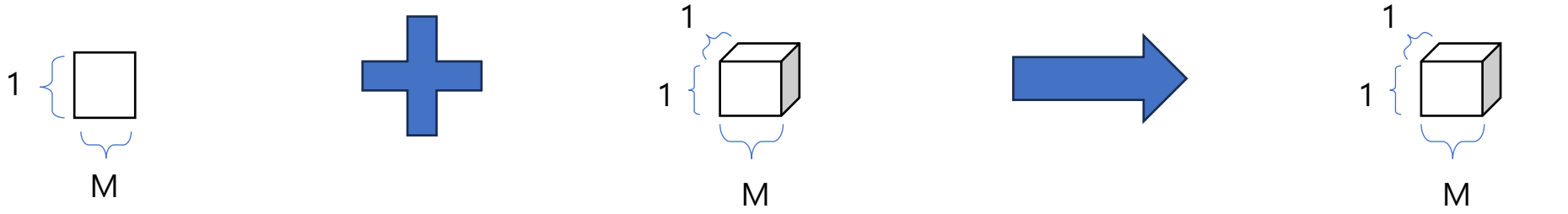


(1, 1, M, 1)

(1, 1, M, 1)

(1, M)

torch.add(y, action)



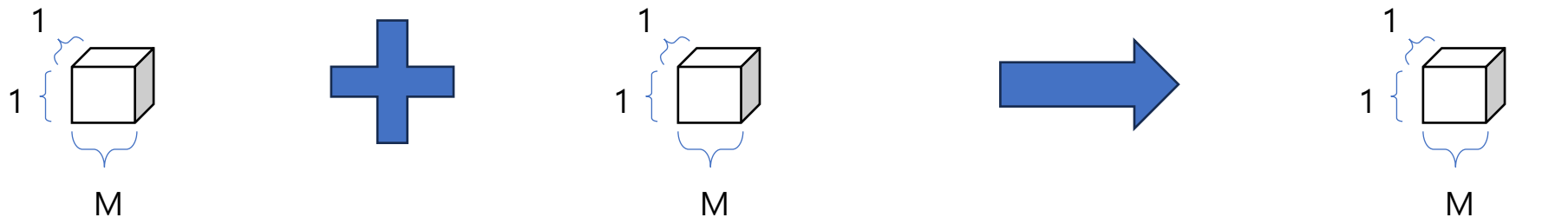
(1, M)

(1, 1, 1 ,M)

(1, 1, 1 ,M)

action

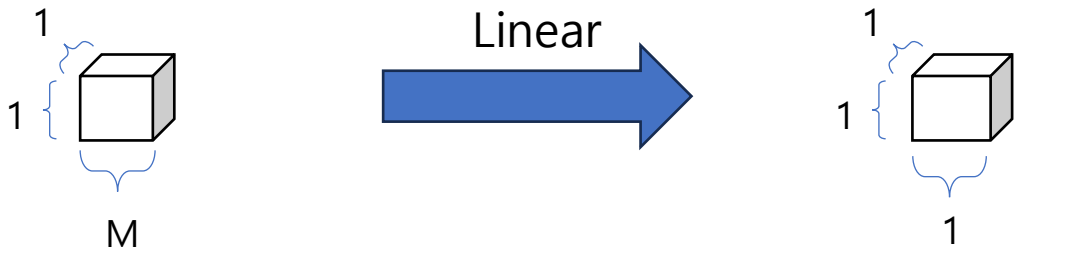
torch.add(y, w)



(1, 1, 1 ,M) (1, 1, 1 ,M) (1, 1, 1 ,M)

previous_action

```
self.fc1 = nn.Linear(self.M, 1)
```



(1, 1, 1 ,M)

(1, 1, 1 ,1)