Early warning signals of financial crises using persistent homology

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The Dotcom Bubble Crash (03/10/2000)

Cause:

Irrational hype & overvaluation of Internet stocks

Impact:

78% NASDAQ collapse; ~$5 trillion in market value erased

eBay: ~91% drop

Yahoo!: ~98% drop

Amazon: ~93% drop

Lehman Brothers Bankruptcy (09/15/2008)

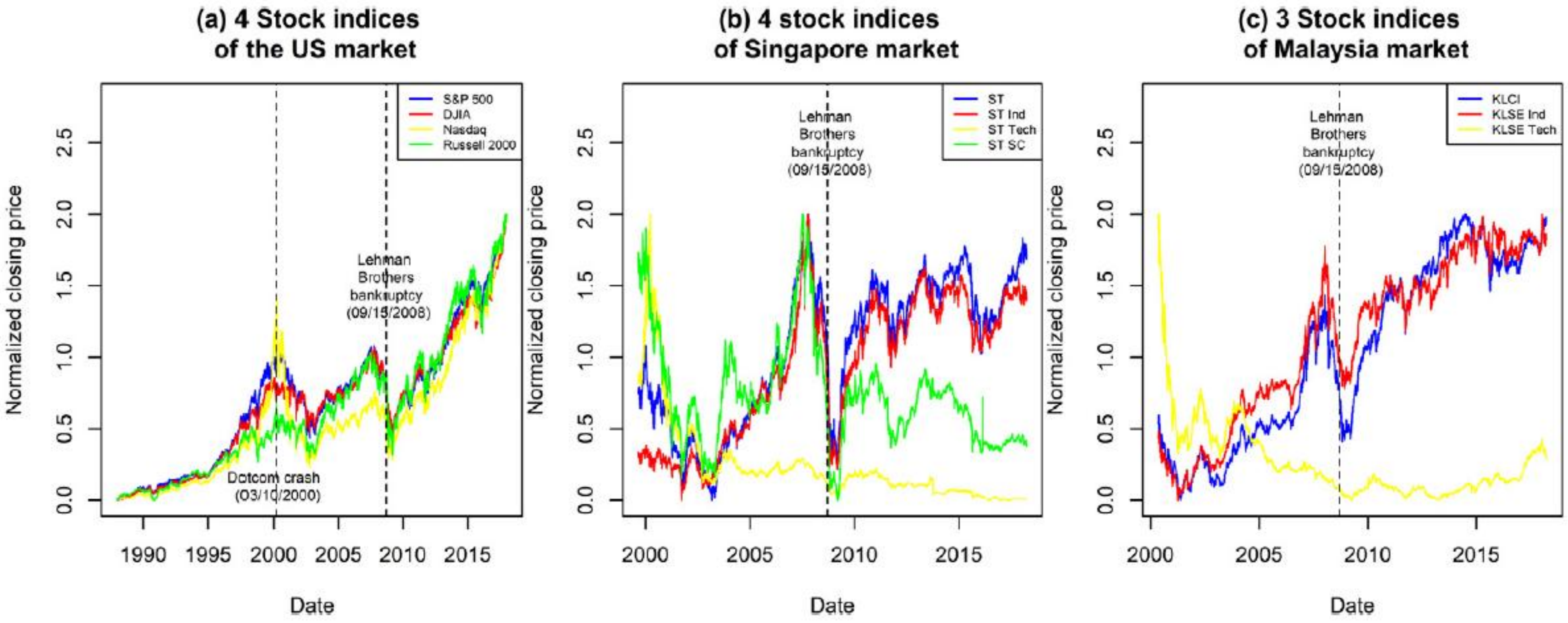
Cause:

Over-leverage on toxic mortgage securities

Impact:

Global equities ~30% drop; credit markets froze

https://www.youtube.com/shorts/m8d23np0oDk



(a) 4 Stock indices of the US market
S&P 500
DJIA
Nasdaq
Russell 2000
Lehman Brothers bankruptcy (09/15/2008)
Dotcom crash (03/10/2000)
Normalized closing price
Date
(b) 4 stock indices of Singapore market
ST
ST Ind
ST Tech
ST SC
Lehman Brothers bankruptcy (09/15/2008)
Normalized closing price
Date
(c) 3 Stock indices of Malaysia market
KLCI
KLSE Ind
KLSE Tech
Lehman Brothers bankruptcy (09/15/2008)
Normalized closing price
Date

Critical Slowing Down (CSD) Theory

- Definition:

As a system nears a tipping point, it recovers more slowly from perturbations

At tipping point, stability collapses, triggering an abrupt change

- Statistical Signs:

- Autocorrelation (ACF1) ↑

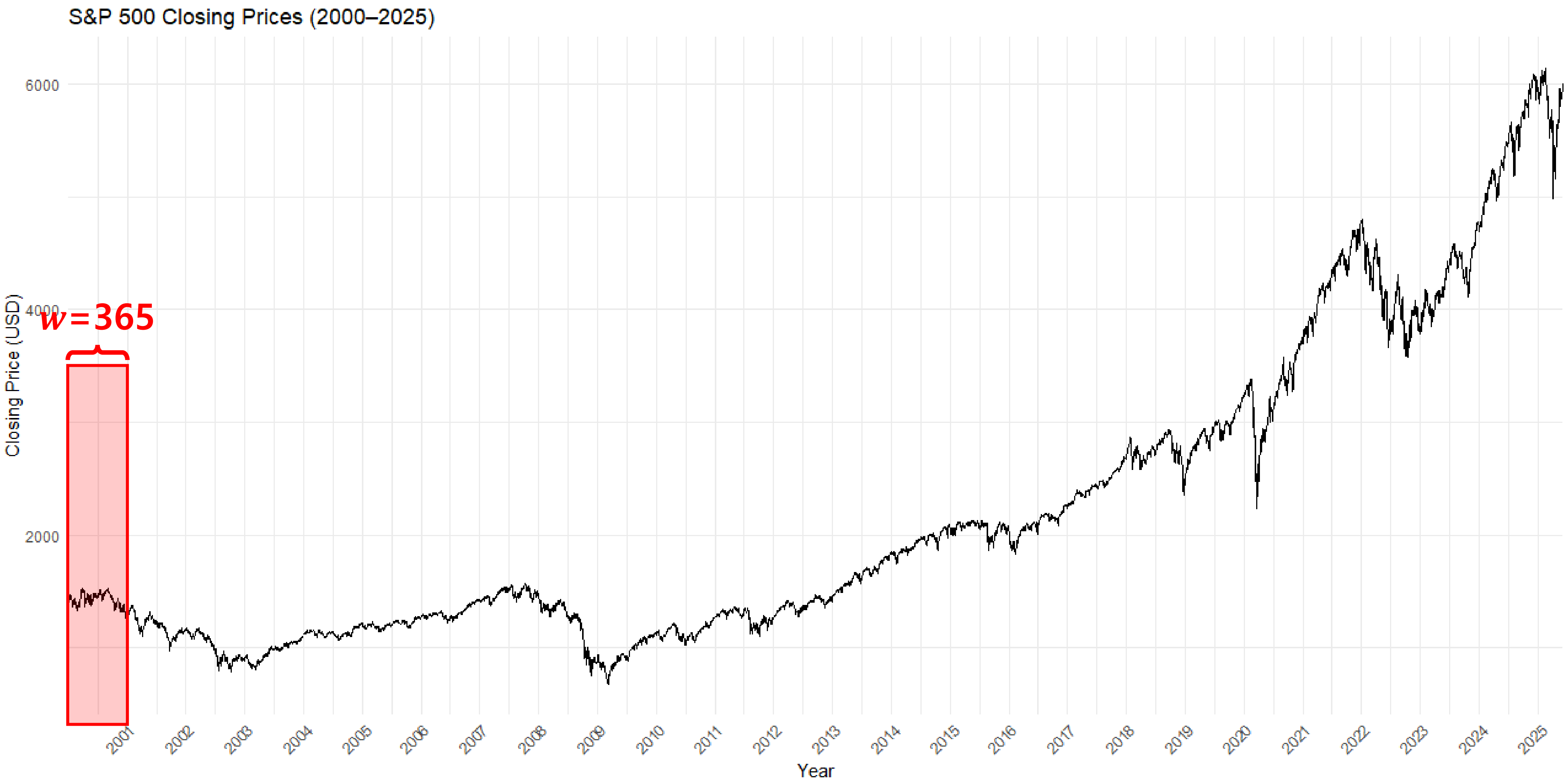
- Variance ↑

- Mean power spectrum (MPS) at low frequencies ↑

- Finance Insight:

Rising CSD metrics show loss of market resilience

Background: Sliding Window



Step 1: Data Collection

- **Dotcom Crash (03/10/2000)**

- US

- **Lehman Brothers Bankruptcy (09/15/2008)**

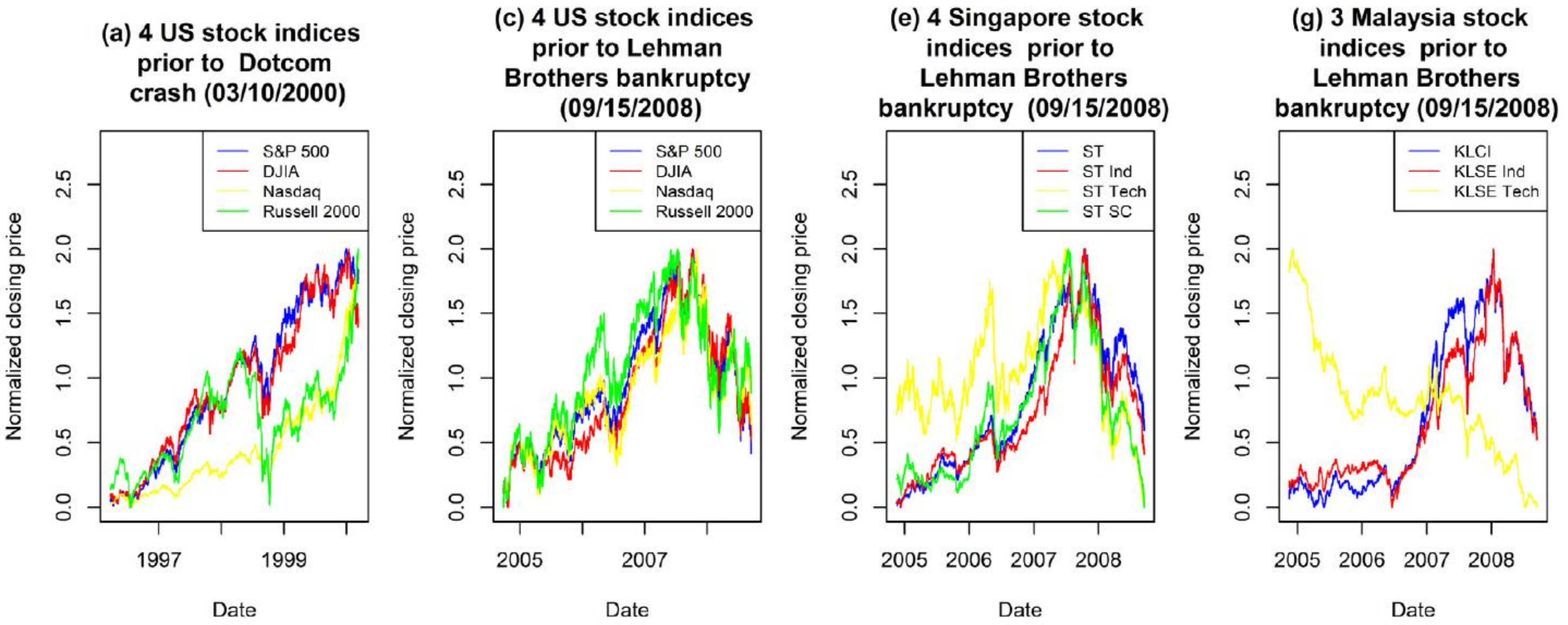
- US
- Singapore
- Malaysia

We use the 1,001 trading days prior to each event.

Step 1: Data Collection

- Collect daily closing prices of d stock indices over the 1,001 trading days prior to each crash.

	1st index (leading companies of **all** sectors)	2nd index (leading companies in the **industrial** sector)	3rd index (leading companies in the **technology** sector)	4th index (leading **small-cap** companies)
US $(d = 4)$	S&P 500	DJIA	Nasdaq	Russel 2000
Singapore $(d = 4)$	ST	ST Ind	ST Tech	ST SC
Malaysia $(d = 3)$	KLCI	KLSE Ind	KLSE Tech	


(a) 4 US stock indices prior to Dotcom crash (03/10/2000)
S&P 500
DJIA
Nasdaq
Russell 2000
Normalized closing price
0.0
0.5
1.0
1.5
2.0
2.5
1997
1999
Date
(c) 4 US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
S&P 500
DJIA
Nasdaq
Russell 2000
Normalized closing price
2005
2007
Date
(e) 4 Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
ST
ST Ind
ST Tech
ST SC
Normalized closing price
2005
2006
2007
2008
Date
(g) 3 Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
KLCI
KLSE Ind
KLSE Tech
Normalized closing price
2005
2006
2007
2008
Date

Step 2: Log-returns Transformation

Compute log-returns

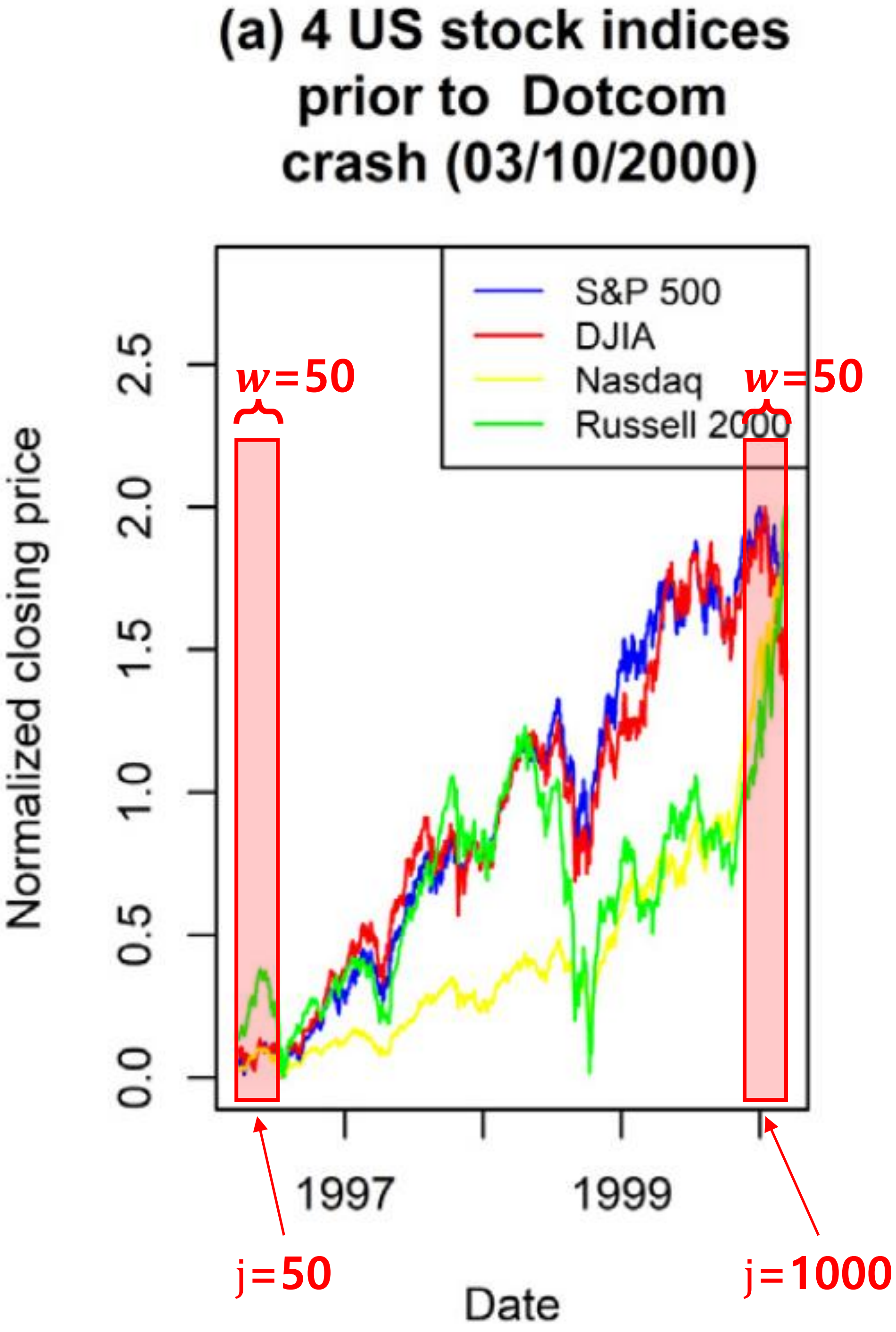
$$x_i(t) = \ln\left(\frac{P_i(t)}{P_i(t-1)}\right)$$

- $x_i(t)$: log-return of index i on day t
- $P_i(t)$: closing price of index i on day t

Step 3: Form Point Cloud Dataset(PCD)

Form point cloud dataset (PCD) of window size $w_1 = 50$ on day $j = 50, 51, \ldots, 1000.$

$$X(j) = \begin{bmatrix} x_1(j-50+1) & x_2(j-50+1) & \cdots & x_d(j-50+1) \\ x_1(j-50+2) & x_2(j-50+2) & & x_d(j-50+2) \\ & \vdots & \ddots & \vdots \\ x_1(j) & x_2(j) & \cdots & x_d(j) \end{bmatrix}$$



(a) 4 US stock indices prior to Dotcom crash (03/10/2000)
S&P 500
DJIA
Nasdaq
Russell 2000
w=50
w=50
Normalized closing price
0.0
0.5
1.0
1.5
2.0
2.5
1997
1999
j=50
j=1000
Date

Step 4: Vietoris-Rips Complex

- Build Vietoris-Rips complexes on log-return point cloud to capture loop structure
- Track appearance (birth) and disappearance (death) of loops across scales
- Use scale parameters

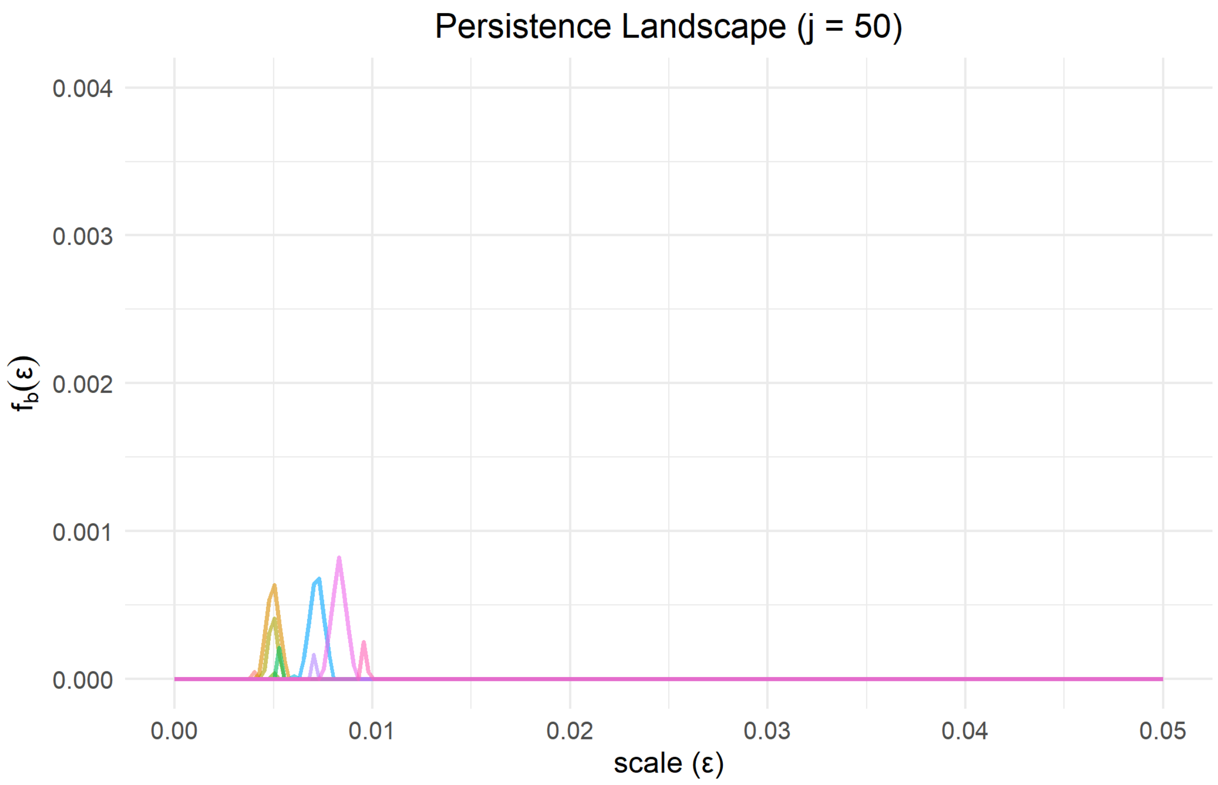
$$0 = \varepsilon_0 < \varepsilon_1 < \cdots < \varepsilon_{max} = 0.05$$

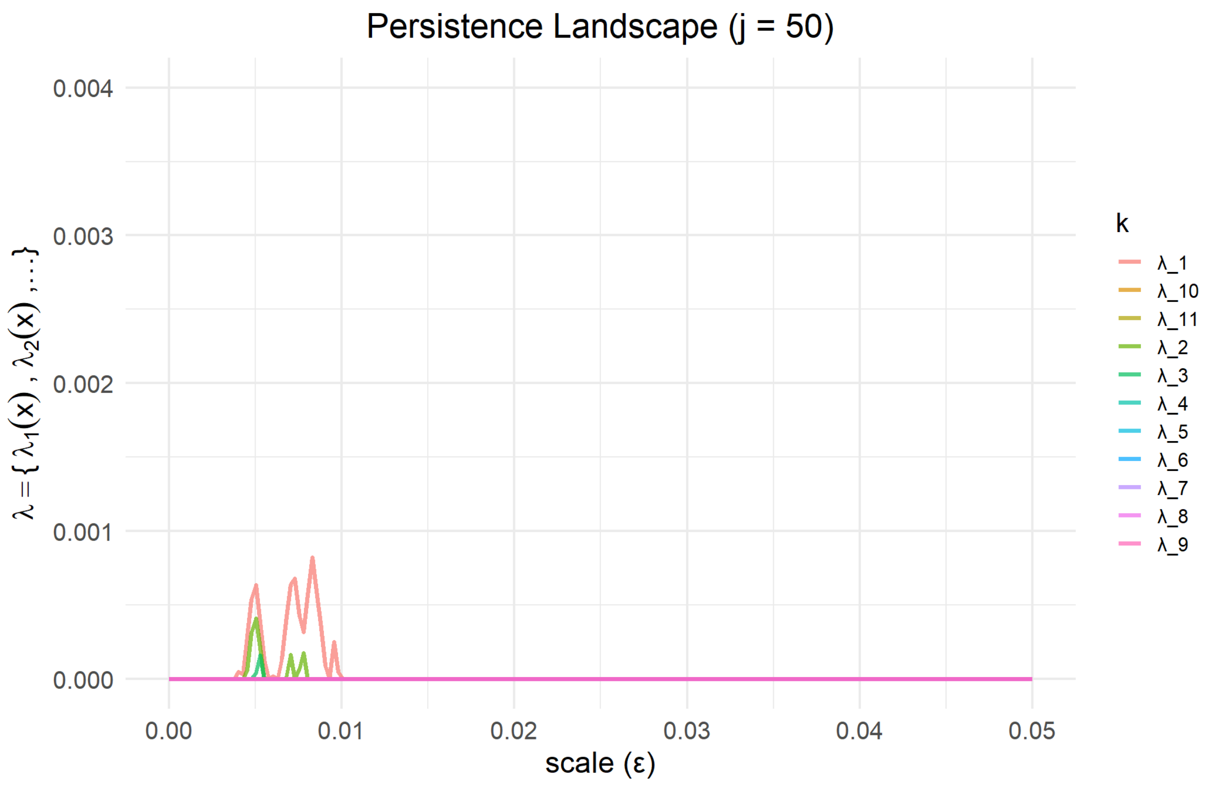
Step 5: Persistence Landscape

- For each birth-death pair $\left(\varepsilon_b^i, \varepsilon_d^i\right)$, define
$$f_{\left(\varepsilon_b^i, \varepsilon_d^i\right)}(x) = \max\{0, \min\{x - \varepsilon_b^i, \varepsilon_d^i - x\}\}$$
- The persistence landscape $\lambda = \{\lambda_1(x), \lambda_2(x), \ldots\}$ is then
$$\lambda_k(x) = k - max\left(\left\{f_{\left(\varepsilon_b^i, \varepsilon_d^i\right)}(x) \middle| \left(\varepsilon_b^i, \varepsilon_d^i\right)\right\}_{i=1}^{n}\right),$$

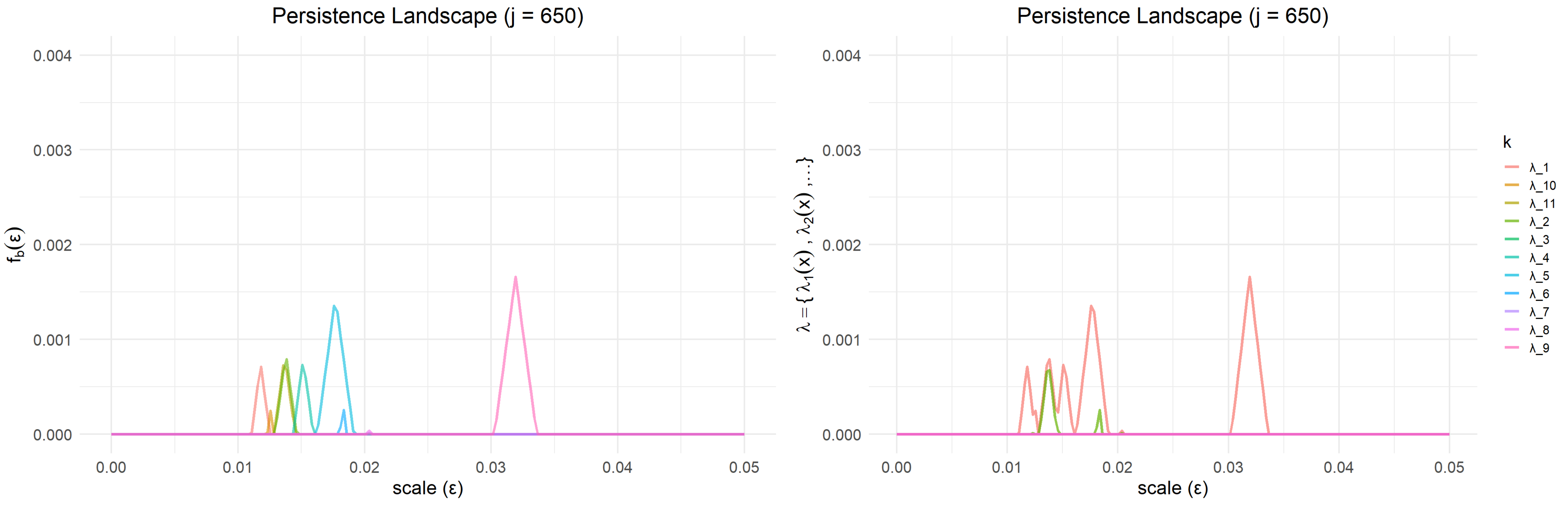
where $k - \max$ picks the k-th largest value at each x.

Persistence Landscape for $j = 50$

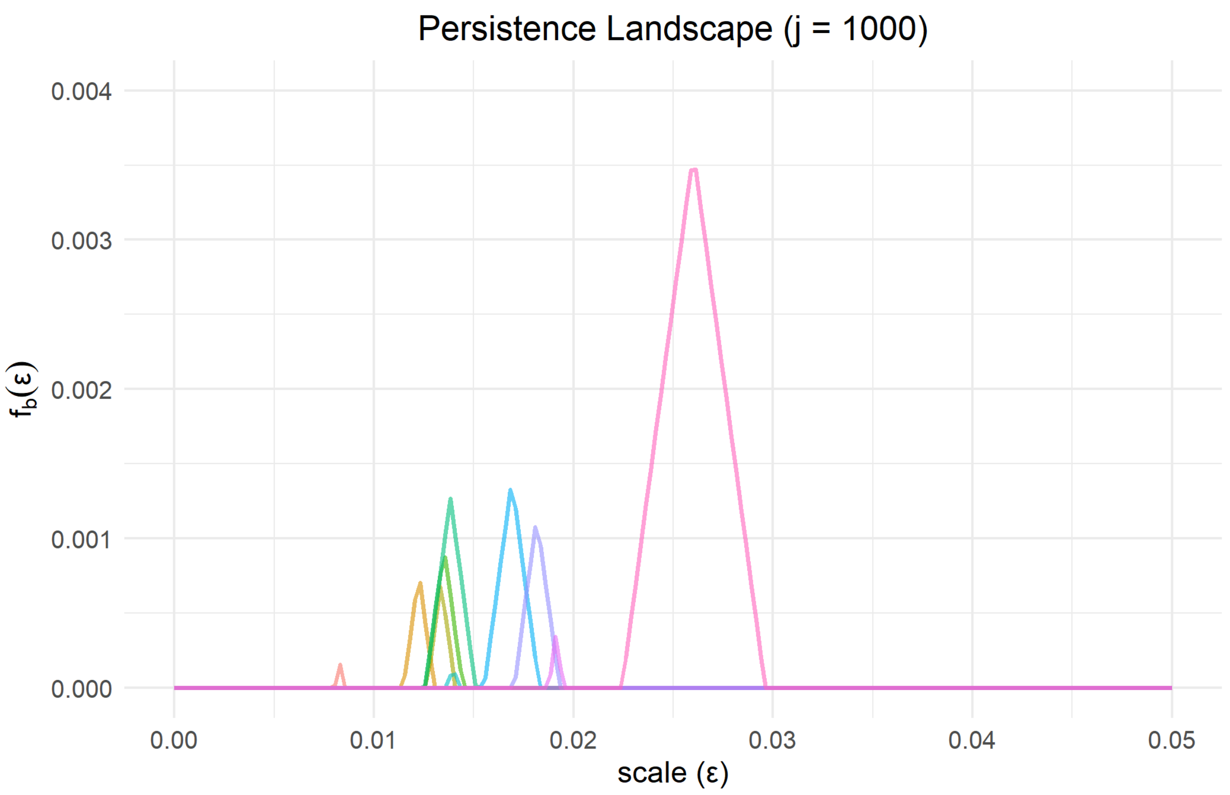


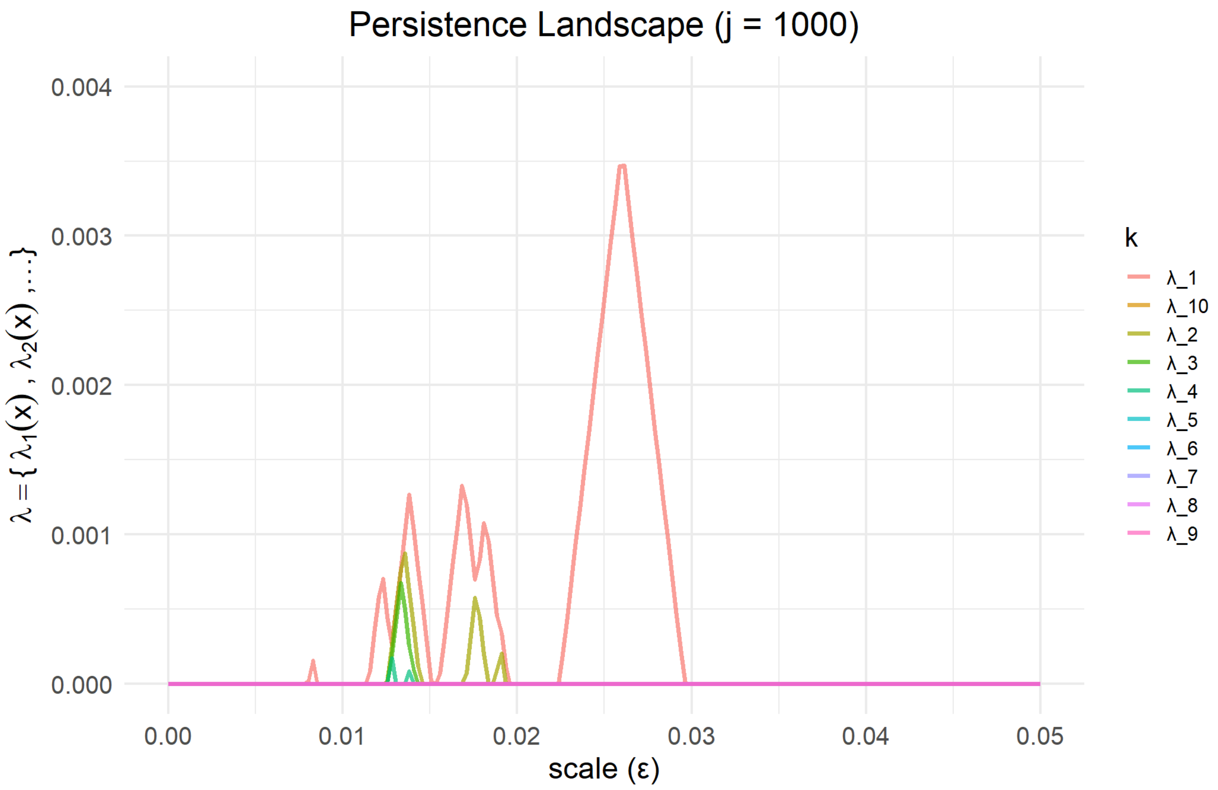


Persistence Landscape for $j = 650$



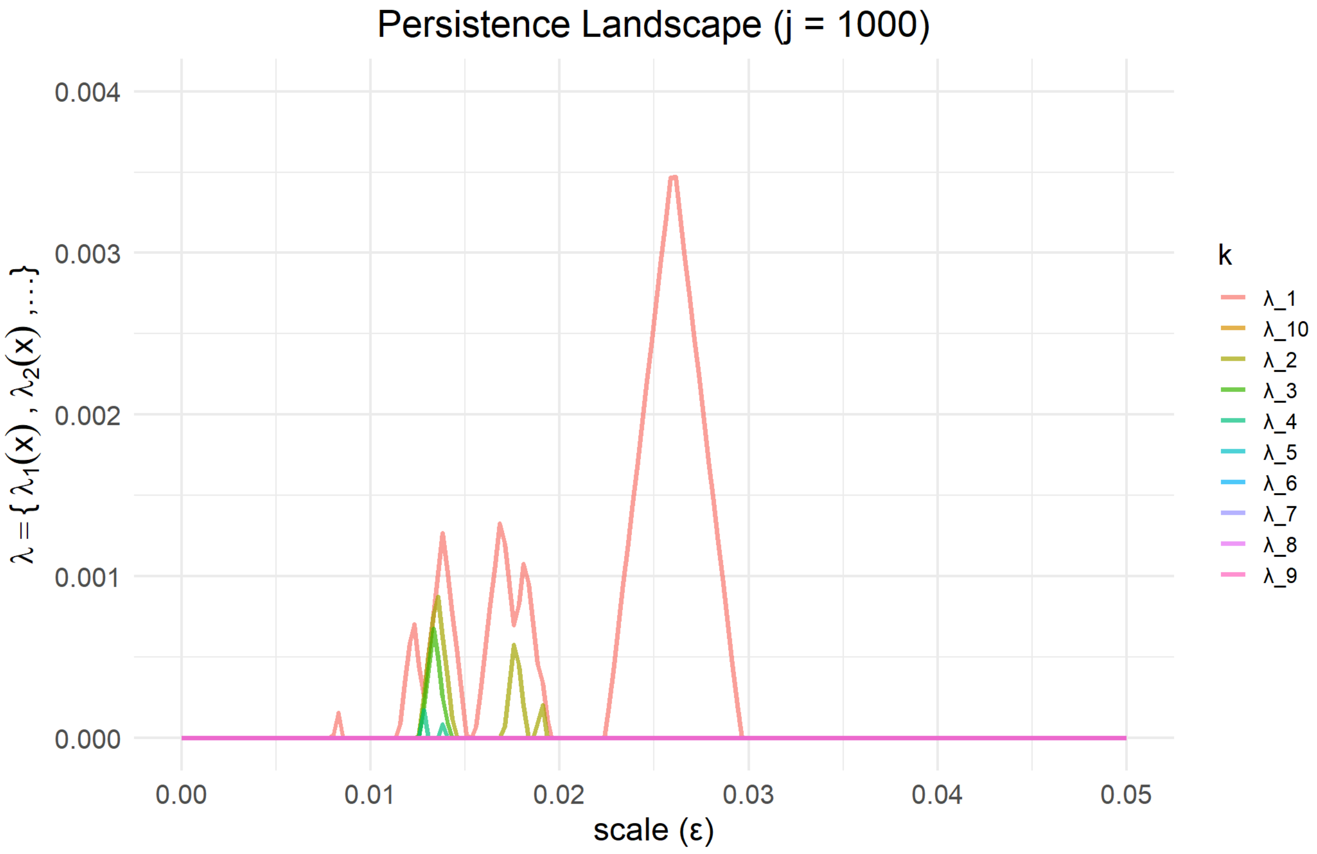
Persistence Landscape for $j = 1000$

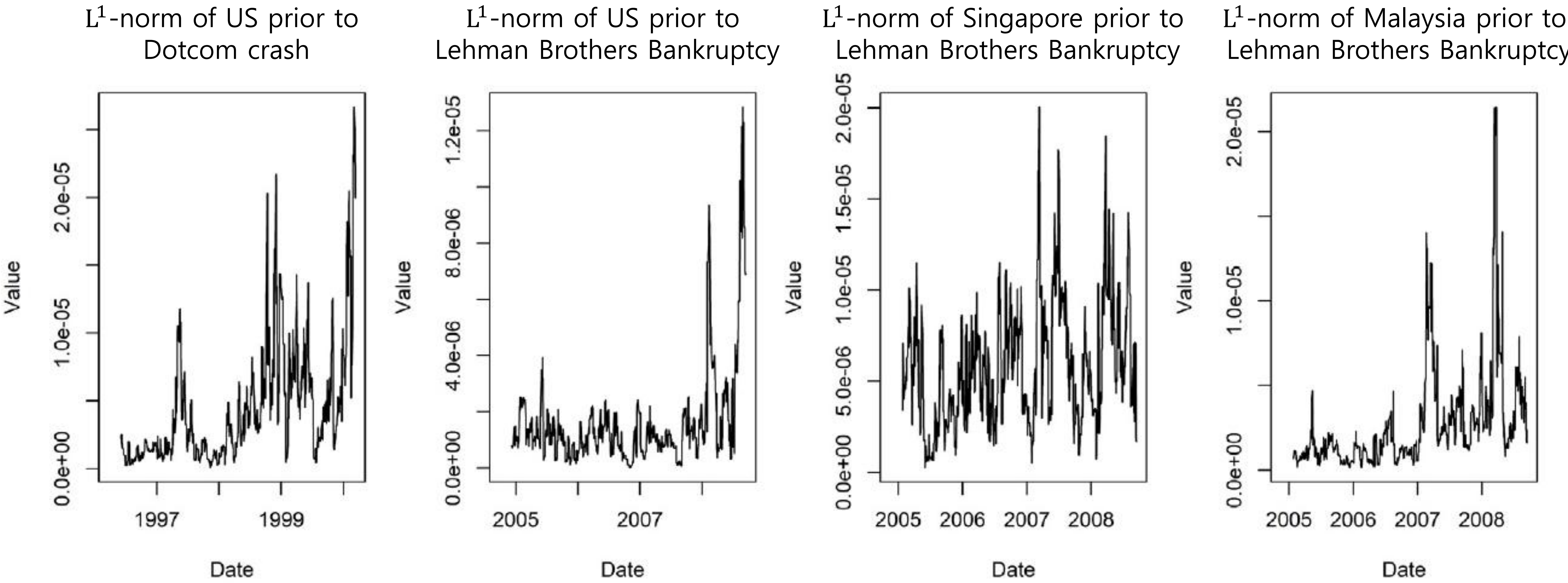




Step 6: L^1-norm

- Compute $\|\lambda\|_1 = \sum_{i=1}^{\infty} \int |\lambda_k(t)|\, dt$




L^1-norm of US prior to Dotcom crash
L^1-norm of US prior to Lehman Brothers Bankruptcy
L^1-norm of Singapore prior to Lehman Brothers Bankruptcy
L^1-norm of Malaysia prior to Lehman Brothers Bankruptcy
Value
Date
0.0e+00
1.0e-05
2.0e-05
1997
1999
4.0e-06
8.0e-06
1.2e-05
2005
2007
5.0e-06
1.5e-05
2006
2008

Step 7: Critical Slowing Down (CSD)

Compute the following CSD Indicators using L^1-norm using sliding window ($w_2 = 250$)

- Autocorrelation function at lag 1 (ACF1)
- Variance (VAR)
- Mean power spectrum (MPS) at low frequencies

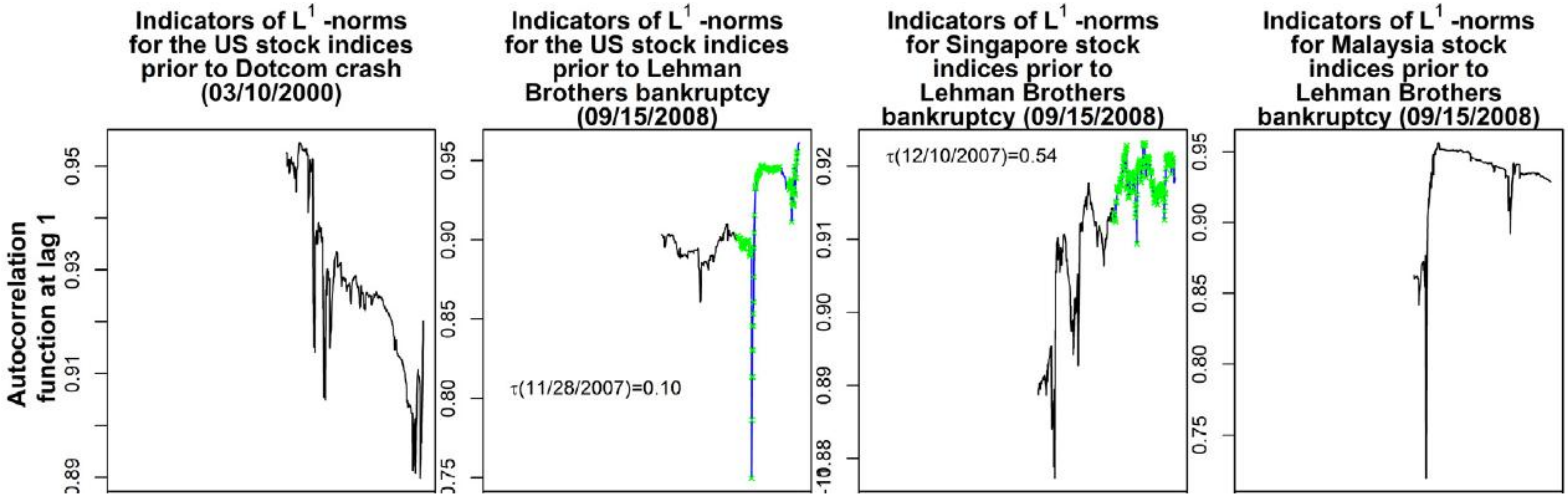
Step 7-1: Autocorrelation function at lag 1 (ACF1)

The ACF1 value at trading day l:

$$acf1(l) = \frac{\rho_1(l)}{var(l)},$$

where

- $\rho_1(l) = \frac{1}{500-1} \sum_{\mathrm{j}=l-500+1}^{l-1} (y(j) - \bar{y}(l))(y(j+1) - \bar{y}(l))$
- $\bar{\mathrm{y}}(l) = \frac{1}{500} \sum_{\mathrm{j}=\mathrm{l}-500+1}^{\mathrm{l}} \mathrm{y}(\mathrm{j})$
- $var(l) = \frac{1}{500-1} \sum_{\mathrm{j}=\mathrm{l}-500+1}^{\mathrm{l}} (\mathrm{y}(\mathrm{j}) - \bar{\mathrm{y}}(\mathrm{l}))^2$



Indicators of L1 -norms for the US stock indices prior to Dotcom crash (03/10/2000)
Indicators of L1 -norms for the US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Autocorrelation function at lag 1
0.89
0.91
0.93
0.95
0.75
0.80
0.85
0.90
0.95
τ(11/28/2007)=0.10
0.88
0.89
0.90
0.91
0.92
τ(12/10/2007)=0.54
0.75
0.80
0.85
0.90
0.95

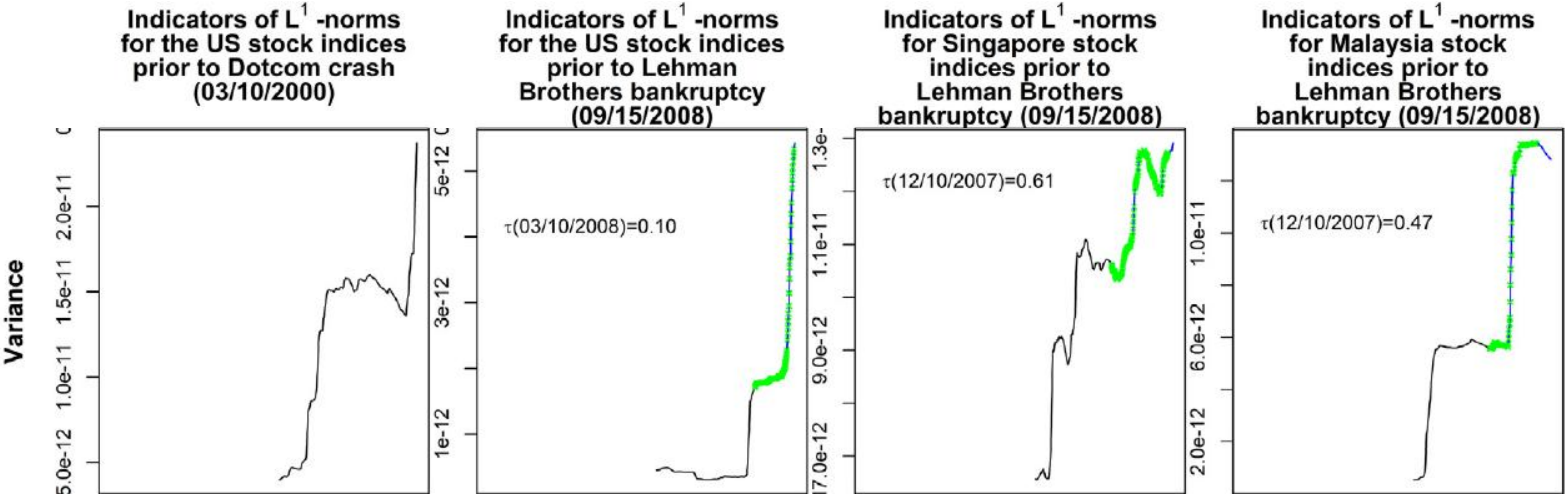
Step 7-2: Variance (VAR)

- The VAR value at trading day l:

$$var(l) = \frac{1}{500-1}\sum_{\mathrm{j=l-500+1}}^{\mathrm{l}} (\mathrm{y(j)} - \bar{\mathrm{y}}(\mathrm{l}))^2$$

where

- $\bar{\mathrm{y}}(l) = \frac{1}{500}\sum_{\mathrm{j=l-500+1}}^{\mathrm{l}} \mathrm{y(j)}$



Indicators of L1 -norms for the US stock indices prior to Dotcom crash (03/10/2000)
Indicators of L1 -norms for the US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Variance
5.0e-12
1.0e-11
1.5e-11
2.0e-11
1e-12
3e-12
5e-12
τ(03/10/2008)=0.10
7.0e-12
9.0e-12
1.1e-11
1.3e-
τ(12/10/2007)=0.61
2.0e-12
6.0e-12
1.0e-11
τ(12/10/2007)=0.47

Step 7-3: Mean power spectrum (MPS) at low frequencies

- **Discrete Fourier Transform:**

$$\mathrm{F_k}(l) = \sum_{\mathrm{n=l-500+1}}^{\mathrm{l}} \mathrm{y(n)}e^{\frac{-2\pi\mathrm{ikn}}{500}}$$

where $k = 1, \ldots, 500$.

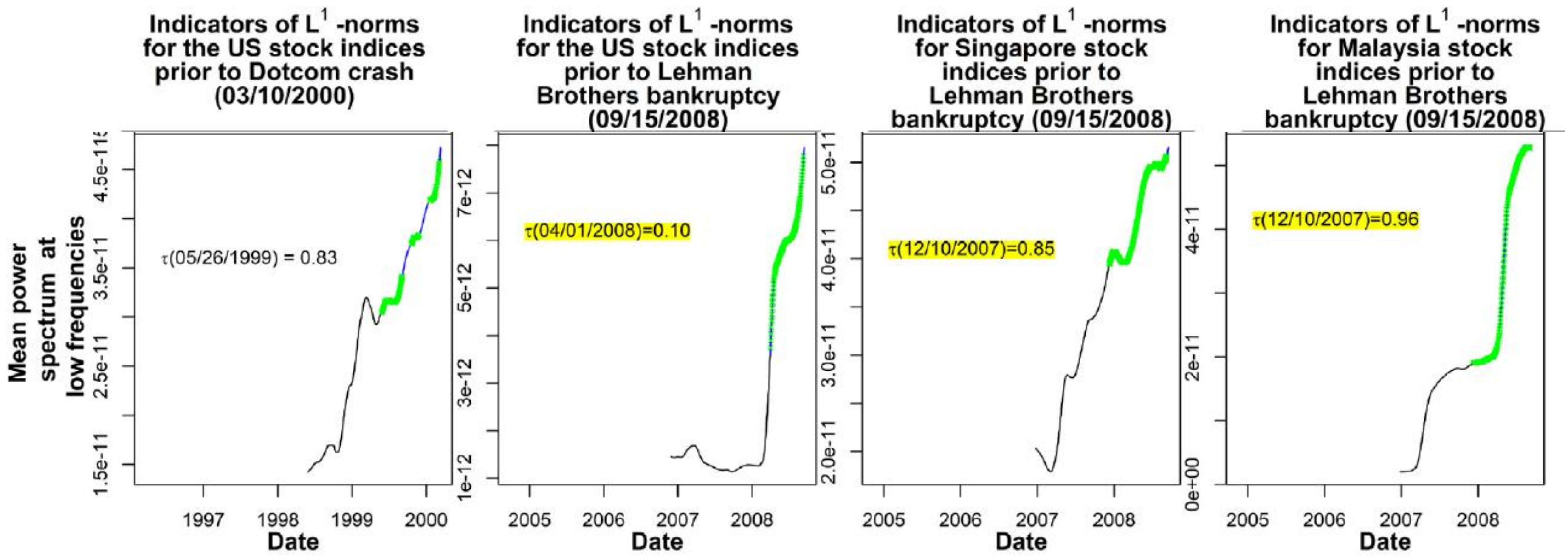
- **Power Spectrum:**

$$PS_{\mathrm{k}}(l) = |\mathrm{F_k}(l)|^2$$

Each $PS_{\mathrm{k}}(l)$ is **normalized** such that its sum is equal to 1.

- **MPS Value:**

$$mps(l) = \frac{1}{\lfloor 500/8 \rfloor - 1} \sum_{\mathrm{k=2}}^{\lfloor 500/8 \rfloor} \mathrm{PS_k(l)}$$



Indicators of L1 -norms for the US stock indices prior to Dotcom crash (03/10/2000)
Mean power spectrum at low frequencies
τ(05/26/1999) = 0.83
1.5e-11
2.5e-11
3.5e-11
4.5e-11
1997
1998
1999
2000
Date
Indicators of L1 -norms for the US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
τ(04/01/2008)=0.10
1e-12
3e-12
5e-12
7e-12
2005
2006
2007
2008
Date
Indicators of L1 -norms for Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
τ(12/10/2007)=0.85
2.0e-11
3.0e-11
4.0e-11
5.0e-11
2005
2006
2007
2008
Date
Indicators of L1 -norms for Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
τ(12/10/2007)=0.96
0e+00
2e-11
4e-11
2005
2006
2007
2008
Date

How can we confirm significant rises in CSD indicators?

- CSD Theory : ACF1, VAR, MPS values should rise before crises

However, this rising trend must be statistically significant

Significance Test: Mann-Kendall Test

- Purpose: Assess significant rising trend in CSD indicators (ACF1, VAR, MPS)
- Window size $w_3 = 250$
- Kendall's τ correlation:

$$\tau_{\mathrm{ACF1}}(\mathrm{m}) = \frac{S(m)}{D(m)}$$

where

- $S(m) = \sum_{\mathrm{p=m-250+1}}^{\mathrm{m-1}} \sum_{\mathrm{q=p+1}}^{\mathrm{m}} sign\big(acf1(q) - acf1(p)\big)$
- $D(m) \; = \left(\frac{1}{2}(250)(249)) - \frac{1}{2}\sum_{\mathrm{r=1}}^{\mathrm{s}}(\alpha_{\mathrm{r}})(\alpha_{\mathrm{r}} - 1)\right)^{1/2} \left(\frac{1}{2}(250)(249))\right)^{1/2}$
- α_{r} is the number of points in the rth tied group

Significance Test: Mann-Kendall Test

H_0: no monotonic trend (rising or falling) in CSD indicator

Under H_0, we have

- $\mathbb{E}[S(m)] = 0$,
- $\sigma^2(\mathrm{m}) = \frac{(250)(249)(505) - \sum_{\mathrm{r}=1}^{\mathrm{s}} (\alpha_{\mathrm{r}})(\alpha_{\mathrm{r}}-1)(2\alpha_{\mathrm{r}}+5)}{18}$

Transform $S(m)$ into a normally distributed $\mathrm{Z}_{\mathrm{obs}}$:

$$\mathrm{Z}_{\mathrm{obs}} = \begin{cases} \frac{\mathrm{S(m)}-1}{\sigma(\mathrm{m})} & \text{if } S(m) > 0 \\ 0 & \text{if } S(m) = 0 \\ \frac{\mathrm{S(m)}+1}{\sigma(\mathrm{m})} & \text{if } S(m) < 0. \end{cases}$$

Significance Test: Mann-Kendall Test

- Two-sided p-value:

$$p - \text{value} \;=\; 2P_{Z\sim N(0,1)}(Z > |\mathrm{Z_{obs}}|)$$

- Criteria for Significant Rising Trend:

If $\tau_{\mathrm{ACF1}}(\mathrm{m}) > 0$ and p-value < 0.05, reject H_0.

Structural Break Test: Chow Test

- H_0: no structural break at time b
- H_1: structural break at time b

- Chow F-test statistics:

$$F_{obs} = \frac{(RSS_1 - (RSS_2 + RSS_3))/k}{(RSS_2 + RSS_3)/(1000 - 549 + 1 - 2k)}$$

- Chow F-test statistics:

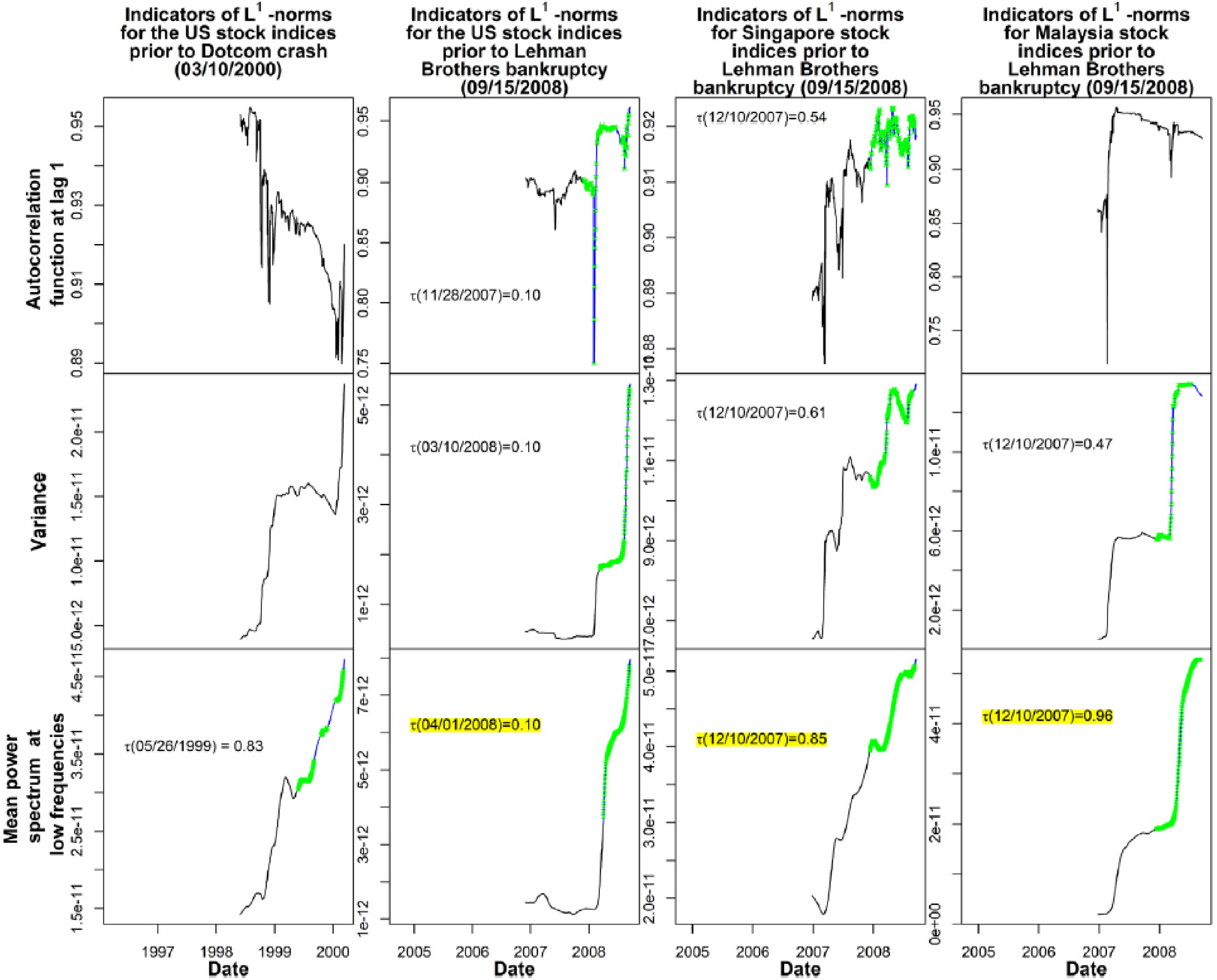
$$F_{obs} = \frac{(RSS_1 - (RSS_2 + RSS_3))/k}{(RSS_2 + RSS_3)/(1000 - 549 + 1 - 2k)}$$

where

- $RSS_1 = (acf1(l) - \hat{\beta}_0^{(1)} - \hat{\beta}_1^{(1)} l)^2$
- $RSS_2 = (acf1(l) - \hat{\beta}_0^{(2)} - \hat{\beta}_1^{(2)} l)^2$
- $RSS_3 = (acf1(l) - \hat{\beta}_0^{(3)} - \hat{\beta}_1^{(3)} l)^2$

- Criteria for Structural Break:

If $P(F > F_{obs}) < 0.05$, reject H_0.



Indicators of L1 -norms for the US stock indices prior to Dotcom crash (03/10/2000)
Indicators of L1 -norms for the US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of L1 -norms for Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Autocorrelation function at lag 1
Variance
Mean power spectrum at low frequencies
τ(11/28/2007)=0.10
τ(12/10/2007)=0.54
τ(03/10/2008)=0.10
τ(12/10/2007)=0.61
τ(12/10/2007)=0.47
τ(05/26/1999) = 0.83
τ(04/01/2008)=0.10
τ(12/10/2007)=0.85
τ(12/10/2007)=0.96
Date
Date
Date
Date

Thresholds for Early Warning Signals

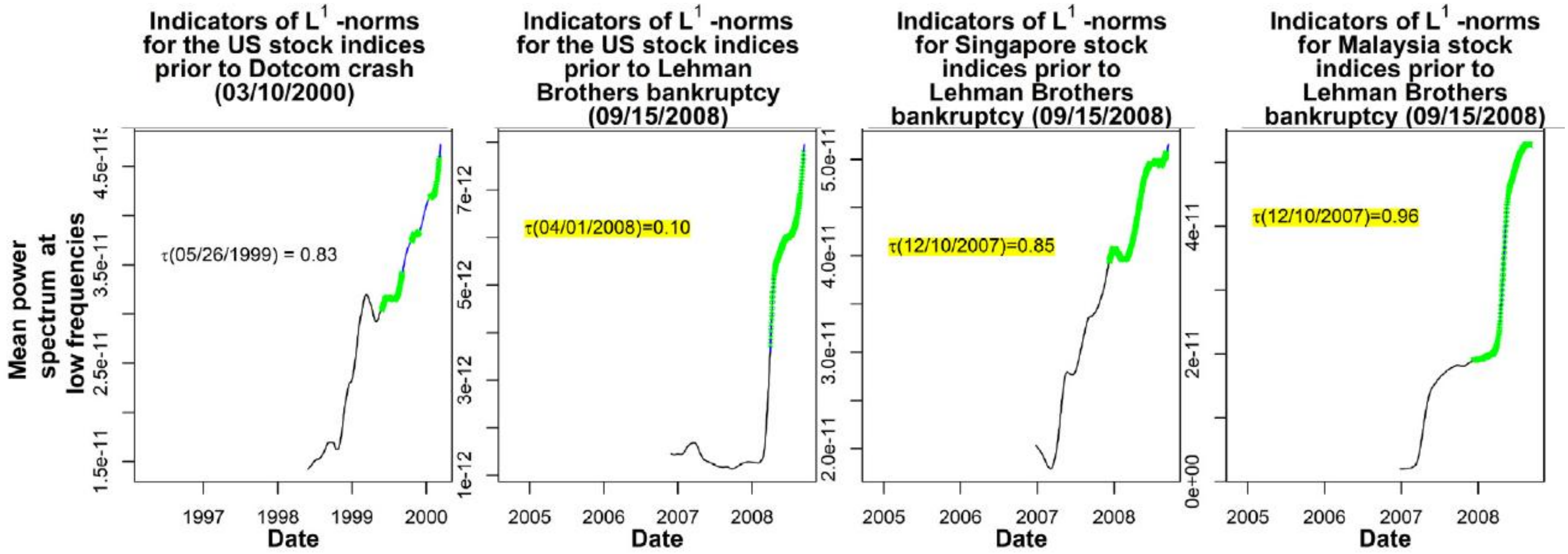
Singapore & Malaysia

- $\{\tau\}_{m=\delta}^{1000}$: significant rising trend for ACF1, VAR, MPS
- Take threshold value T at the first breakpoint within $\{\tau\}_{m=\delta}^{1000}$

US

- Take threshold value $T = \min\{T_{Dotcom}, T_{Lehman}\}$

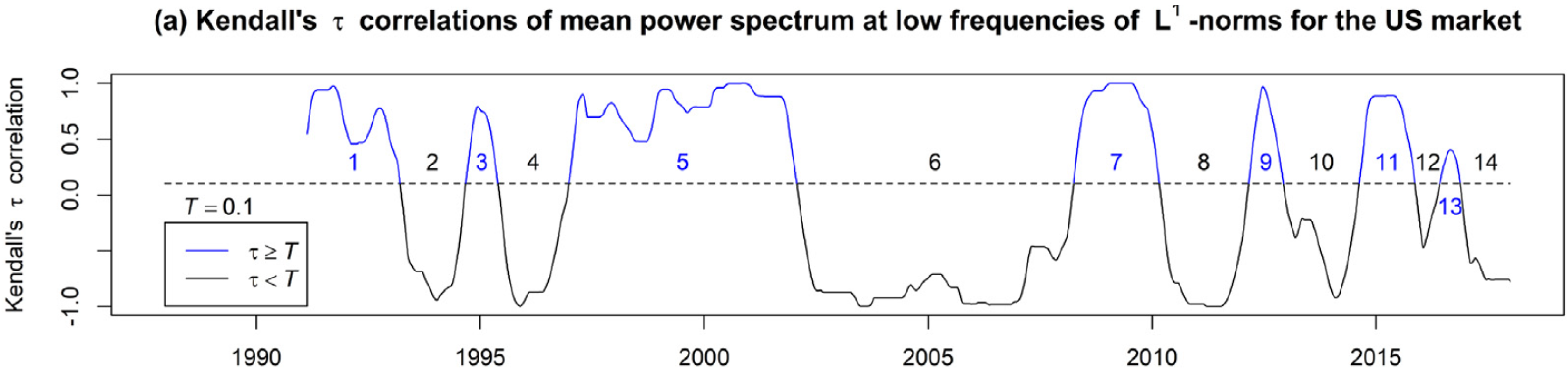
where T_{Dotcom} and T_{Lehman} are determined as in Singapore & Malaysia



$T_{US} = 0.10$

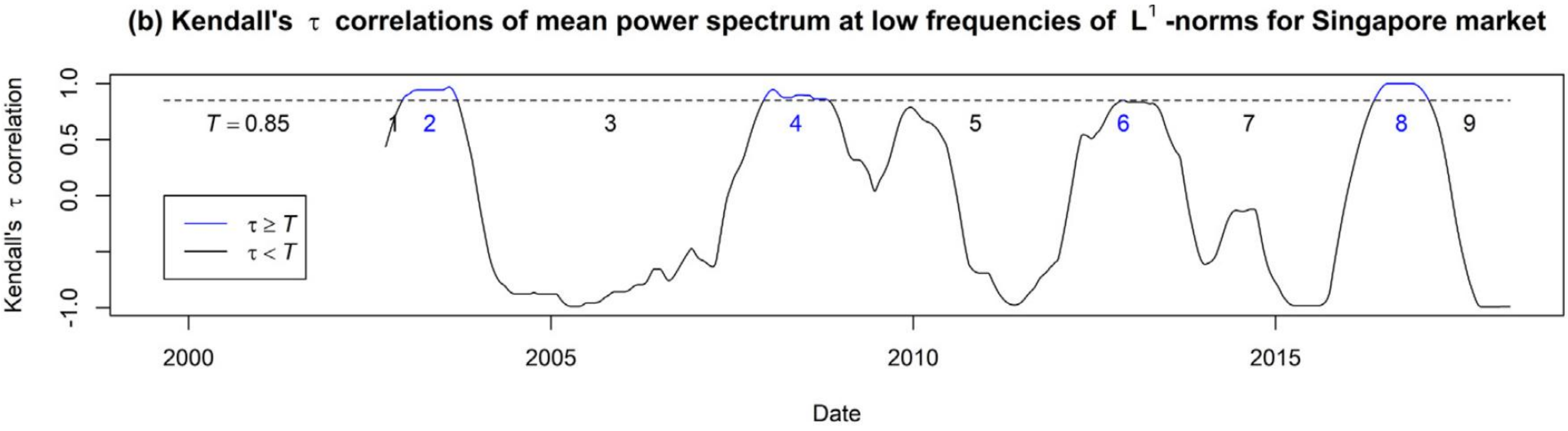
$T_{Singapore} = 0.85$ $T_{Malaysia} = 0.96$

US



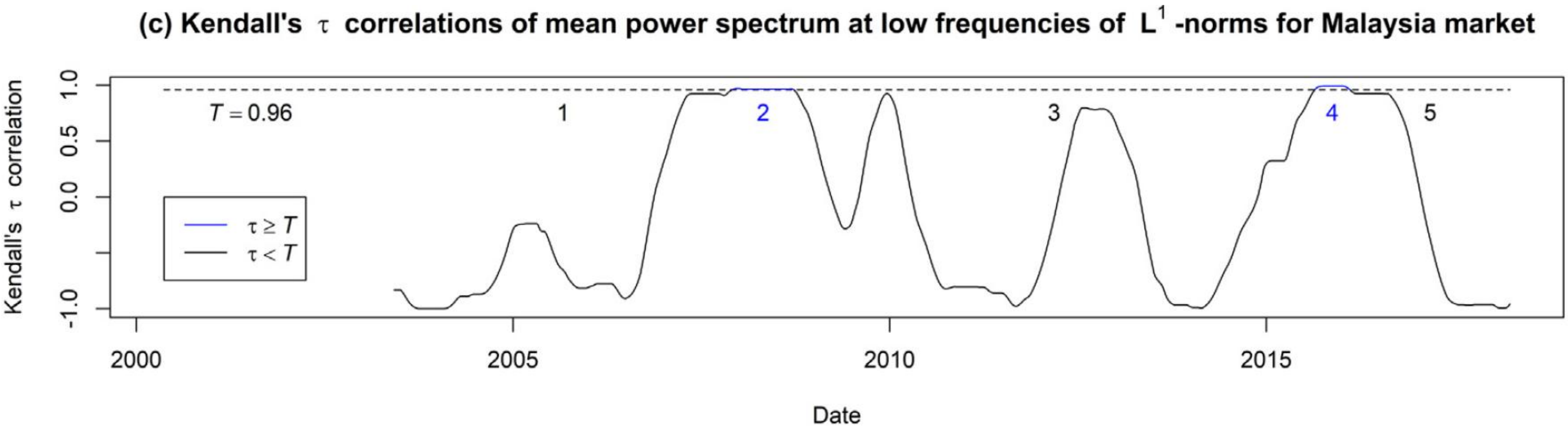
(a) Kendall's τ correlations of mean power spectrum at low frequencies of L^1 -norms for the US market
Kendall's τ correlation
1.0
0.5
0.0
-1.0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
T = 0.1
τ ≥ T
τ < T
1990
1995
2000
2005
2010
2015

Singapore



(b) Kendall's τ correlations of mean power spectrum at low frequencies of L1 -norms for Singapore market
Kendall's τ correlation
1.0
0.5
0.0
-1.0
T = 0.85
1
2
3
4
5
6
7
8
9
τ ≥ T
τ < T
2000
2005
2010
2015
Date

Malaysia



(c) Kendall's τ correlations of mean power spectrum at low frequencies of L^1 -norms for Malaysia market
Kendall's τ correlation
1.0
0.5
0.0
-1.0
T = 0.96
1
2
3
4
5
τ ≥ T
τ < T
2000
2005
2010
2015
Date

Financial Crises

US	Singapore	Malaysia
1. The global mini-crash caused by the 1997 Asian economic crisis (10/27/1997) 2. Dotcom crash (03/10/2000) 3. September 11 attack (09/11/2001) 4. 2002 stock market downturn (10/09/2002) 5. Chinese stock bubble of 2007 (02/27/2007) 6. US bear market of 2007 (10/11/2007) 7. Lehman Brothers Bankruptcy (09/15/2008) 8. 2009 Dubai debt standstill (11/27/2009) 9. 2010 flash crash (03/06/2010) 10. 2015-2016 Chinese stock market crash (06/12/2015) 11. 2015-2016 US stock market sell-off (08/18/2015)	1. Chinese stock bubble of 2007 (02/27/2007) 2. US bear market of 2007 (10/11/2007) 3. Lehman Brothers Bankruptcy (09/15/2008) 4. 2009 Dubai debt standstill (11/27/2009) 5. 2015-2016 Chinese stock market crash (06/12/2015)	1. US bear market of 2007 (10/11/2007) 2. Lehman Brothers Bankruptcy (09/15/2008) 3. 2009 Dubai debt standstill (11/27/2009)

Event Classification

- True Positive (EWS): Signal & Crisis
- False Alarm (FA): Signal & No Crisis
- False Negative (FN): No Signal & Crisis
- True Negative (TN): No Signal & No Crisis

Performance Metrics

Let

- A = # of True Positive (EWS)
- B = # of False Alarm (FA)
- C = # of False Negative (FN)
- D = # of True Negative (TN)

- Probability of Successful Anticipation:

$$\frac{A + D}{A + B + C + D} \times 100\%$$

- Probability of Erroneous Anticipation:

$$\frac{B + C}{A + B + C + D} \times 100\%$$

MPS Results on PL L1-Norm

Market	Score name	Method
		MPS of the $\boldsymbol{L^1}$-norms
US	Probability of successful anticipation (%)	60
	Probability of erroneous anticipation (%)	40
Singapore	Probability of successful anticipation (%)	30
	Probability of erroneous anticipation (%)	70
Malaysia	Probability of successful anticipation (%)	40
	Probability of erroneous anticipation (%)	60

Residual Time Series

- Use only the first stock index

	1st index (leading companies of **all** sectors)	~~2nd index (leading companies in the **industrial** sector)~~	~~3rd index (leading companies in the **technology** sector)~~	~~4th index (leading **small-cap** companies)~~
US $(d = 4)$	S&P 500	~~DJIA~~	~~Nasdaq~~	~~Russel 2000~~
Singapore $(d = 4)$	ST	~~ST Ind~~	~~ST Tech~~	~~ST SC~~
Malaysia $(d = 3)$	KLCI	~~KLSE Ind~~	~~KLSE Tech~~	

Residual Time Series

- $\{x_i(t)\}_{t=1}^{1000}$: closing prices before financial crisis

- **Smoothed index**:

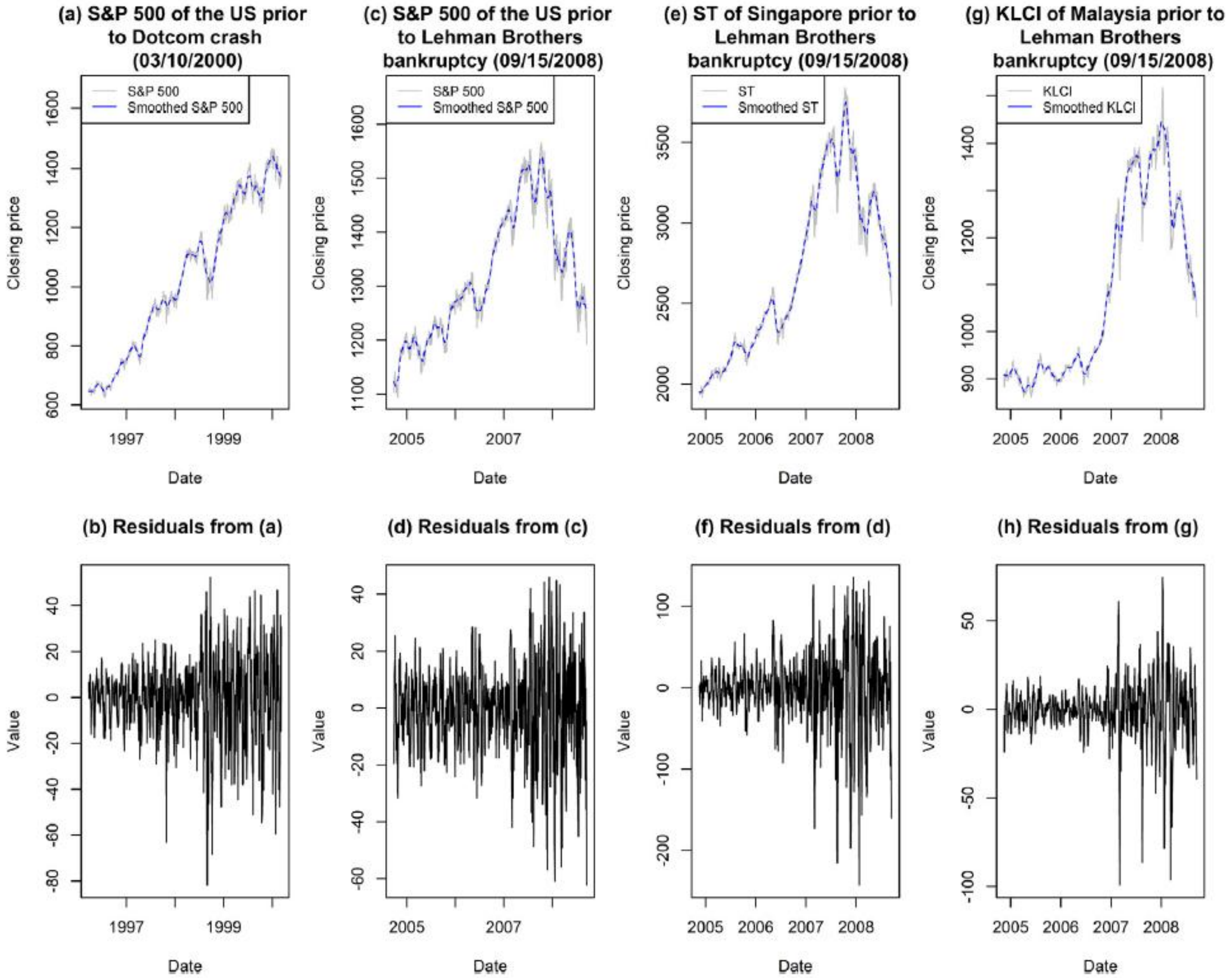
$$y_i(t) = \frac{\sum_{s=1}^{1000} K\left(\frac{t-s}{h}\right) x_i(s)}{\sum_{s=1}^{1000} K\left(\frac{t-s}{h}\right)}, \quad K(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}, \; h = 25$$

- t: target trading day
- s: index over all days in window
- h: bandwidth (25 days)
- $K(u)$: Gaussian kernel weighting function

- **Residual time series**:

$$res_i(t) = x_i(t) - y_i(t)$$

Res


(a) S&P 500 of the US prior to Dotcom crash (03/10/2000)
S&P 500
Smoothed S&P 500
Closing price
Date
(c) S&P 500 of the US prior to Lehman Brothers bankruptcy (09/15/2008)
S&P 500
Smoothed S&P 500
(e) ST of Singapore prior to Lehman Brothers bankruptcy (09/15/2008)
ST
Smoothed ST
(g) KLCI of Malaysia prior to Lehman Brothers bankruptcy (09/15/2008)
KLCI
Smoothed KLCI
(b) Residuals from (a)
(d) Residuals from (c)
(f) Residuals from (d)
(h) Residuals from (g)
Value
Date

Residual Time Series EWS Workflow

1. Compute Residuals $\{res_i(t)\}_{t=1}^{1000}$ for $i = 1$
2. Compute CSD indicators

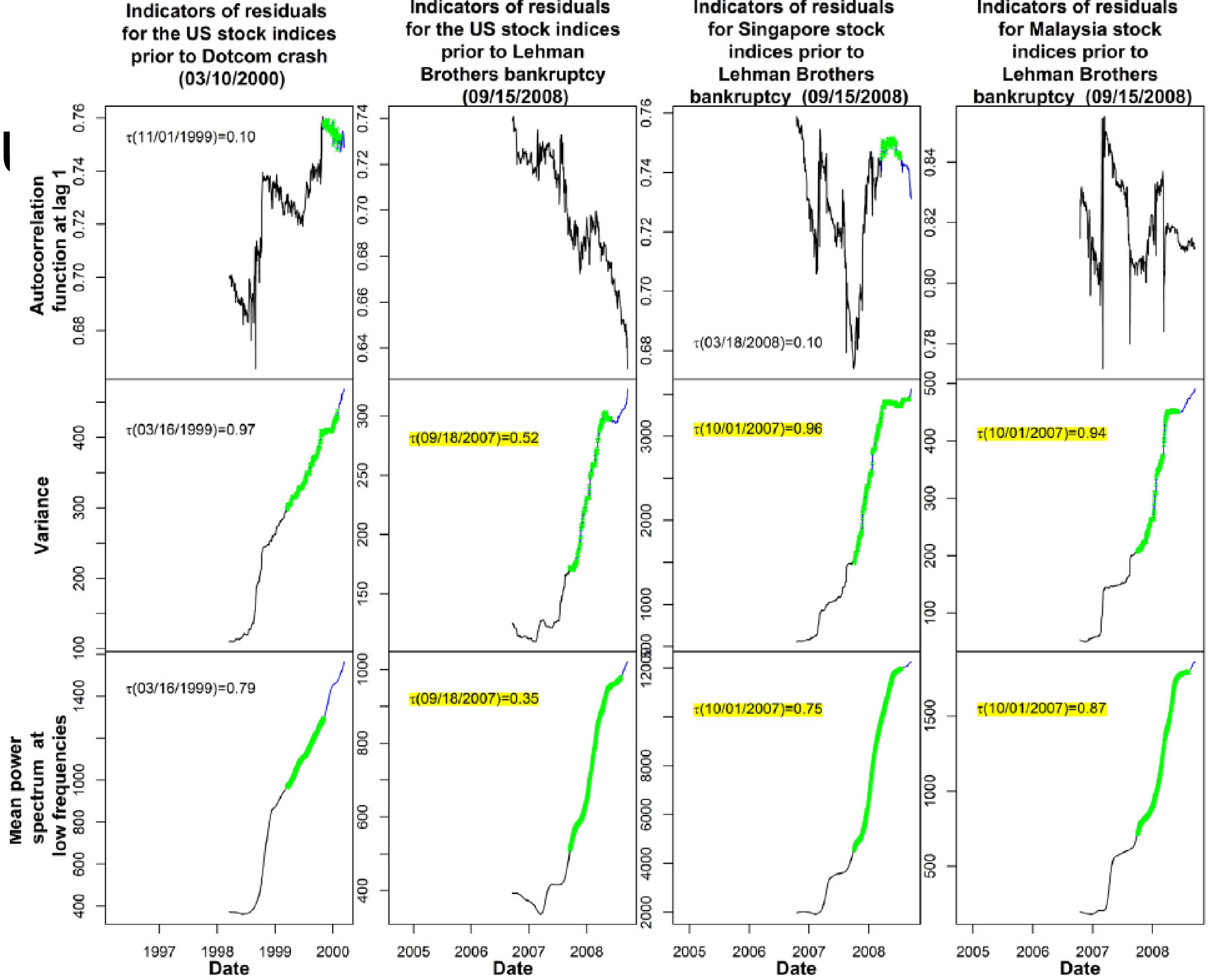
ACF1, Variance, MPS on $\{res_i(t)\}_{t=1}^{1000}$ with $w_2 = 500$

3. Significance Test

Mann-Kendall test with w_3=250

4. Structural Break Test (Chow test)
5. Threshold Application

Resu



Indicators of residuals for the US stock indices prior to Dotcom crash (03/10/2000)
Indicators of residuals for the US stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of residuals for Singapore stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Indicators of residuals for Malaysia stock indices prior to Lehman Brothers bankruptcy (09/15/2008)
Autocorrelation function at lag 1
Variance
Mean power spectrum at low frequencies
τ(11/01/1999)=0.10
τ(03/18/2008)=0.10
τ(03/16/1999)=0.97
τ(09/18/2007)=0.52
τ(10/01/2007)=0.96
τ(10/01/2007)=0.94
τ(03/16/1999)=0.79
τ(09/18/2007)=0.35
τ(10/01/2007)=0.75
τ(10/01/2007)=0.87
Date

Overall Results

Table 12
Summary for the classification scores obtained of the L^1-norms and the residuals.

Market	Score name	Method		
		MPS of the $\boldsymbol{L}^1$-norms	VAR of theresiduals	MPS of the residuals
US	Probability of successful anticipation (%)	60	55	**64**
	Probability of erroneous anticipation (%)	40	45	**36**
Singapore	Probability of successful anticipation (%)	30	20	**38**
	Probability of erroneous anticipation (%)	70	80	**62**
Malaysia	Probability of successful anticipation (%)	40	50	**67**
	Probability of erroneous anticipation (%)	60	50	**33**

Overall Results

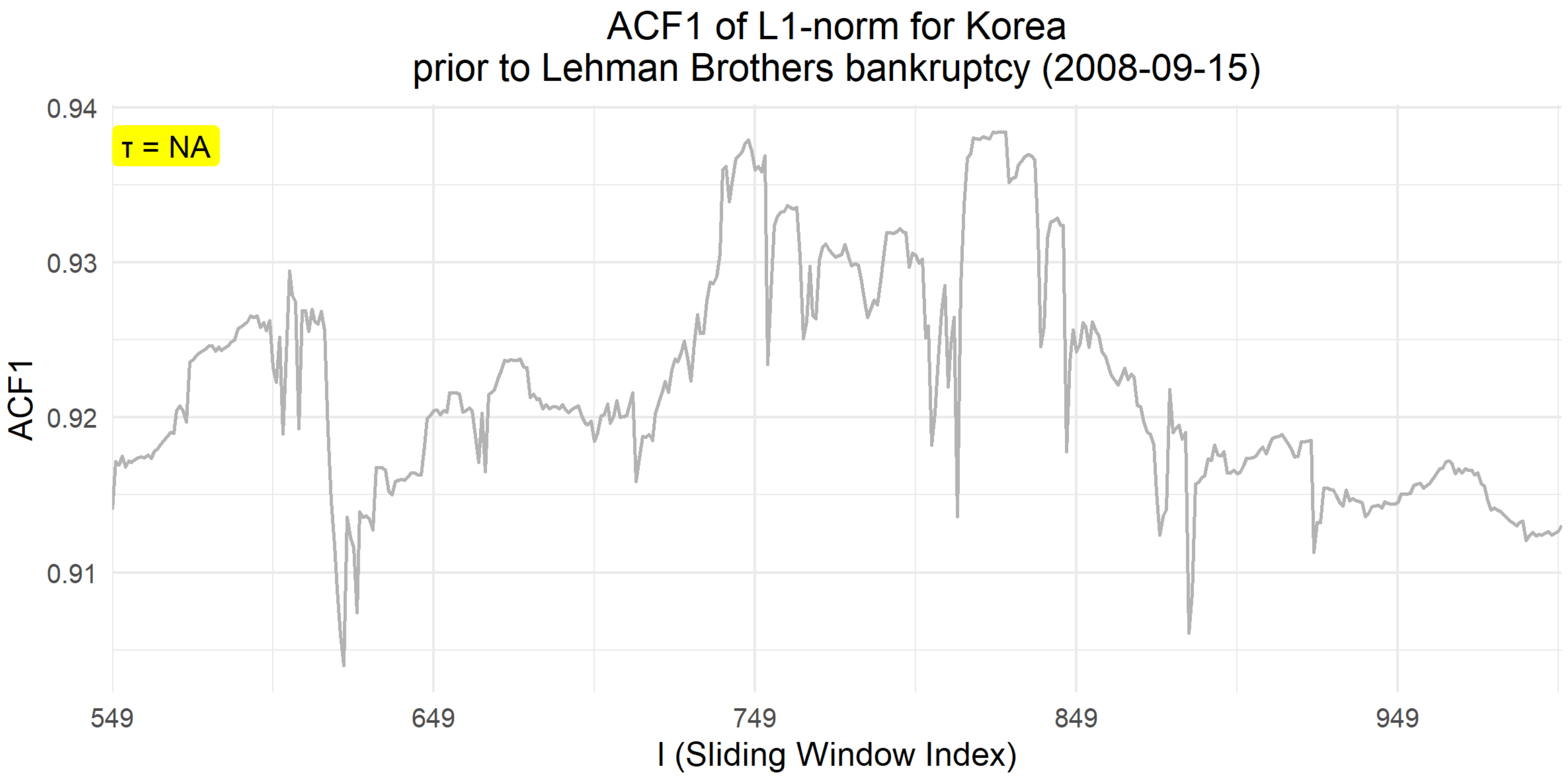
- The US's very low threshold yielded more EWS detections, whereas higher thresholds in Singapore and Malaysia led to fewer signals.
- Malaysia's weaker L1-norm performance likely stems from using only 3 indices versus 4 in the US and Singapore.
- Inappropriate window sizes for Singapore and Malaysia further degraded results, underscoring parameter sensitivity.

Extending to the Korean Market

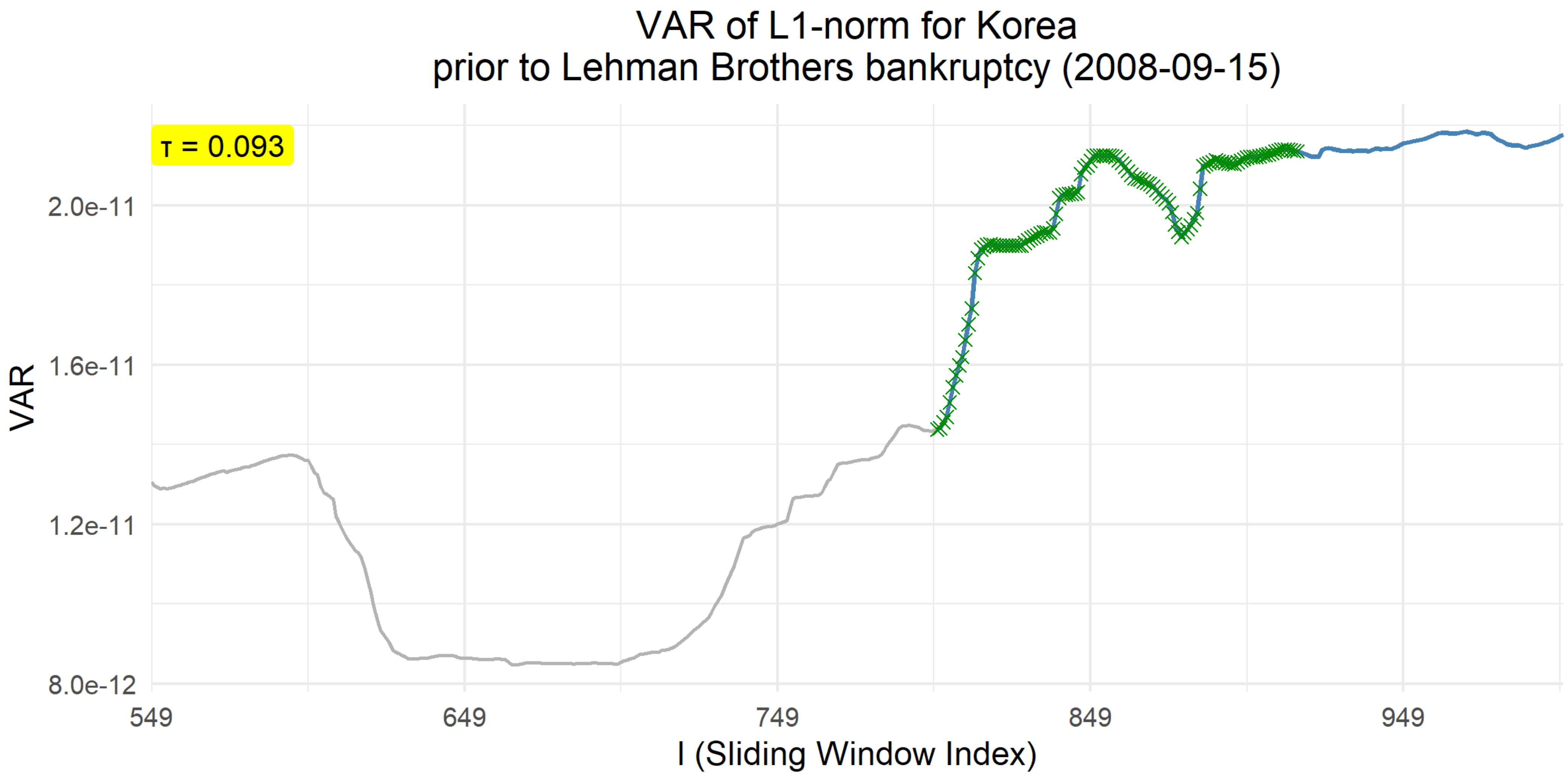
	1st index (leading companies of **all** sectors)	2nd index (leading companies in the **industrial** sector)	3rd index (leading companies in the **technology** sector)	4th index (leading **small-cap** companies)
US $(d = 4)$	S&P 500	DJIA	Nasdaq	Russel 2000
Singapore $(d = 4)$	ST	ST Ind	ST Tech	ST SC
Malaysia $(d = 3)$	KLCI	KLSE Ind	KLSE Tech	
Korea $(d = 4)$	KOSPI 200	KOSPI Manufacturing	KOSPI Electronics	Kosdaq

Key Financial Crises Analyzed: Lehman Brothers Bankruptcy (09/15/2008)

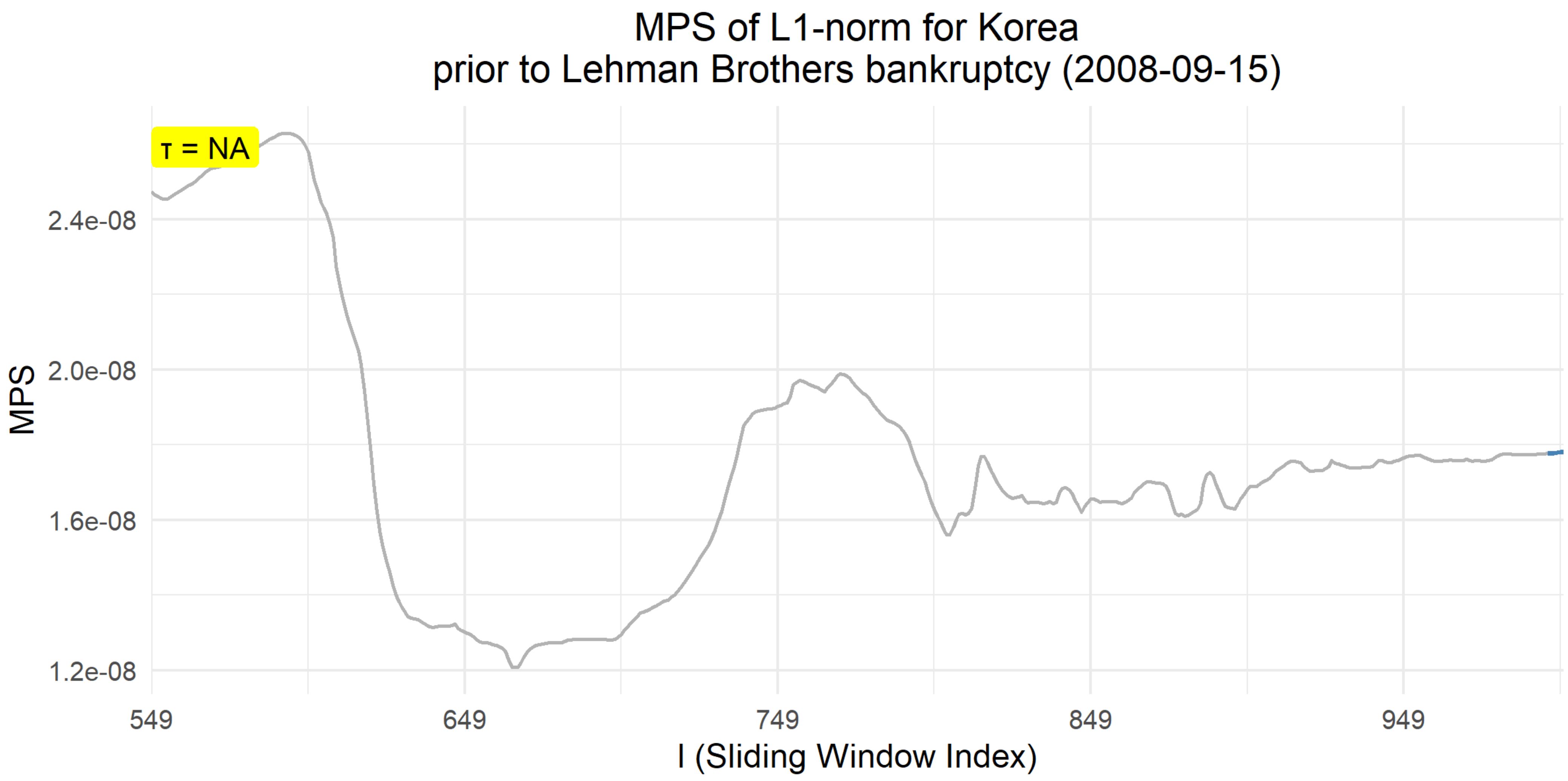
ACF1 on L1-norm - Korean Market



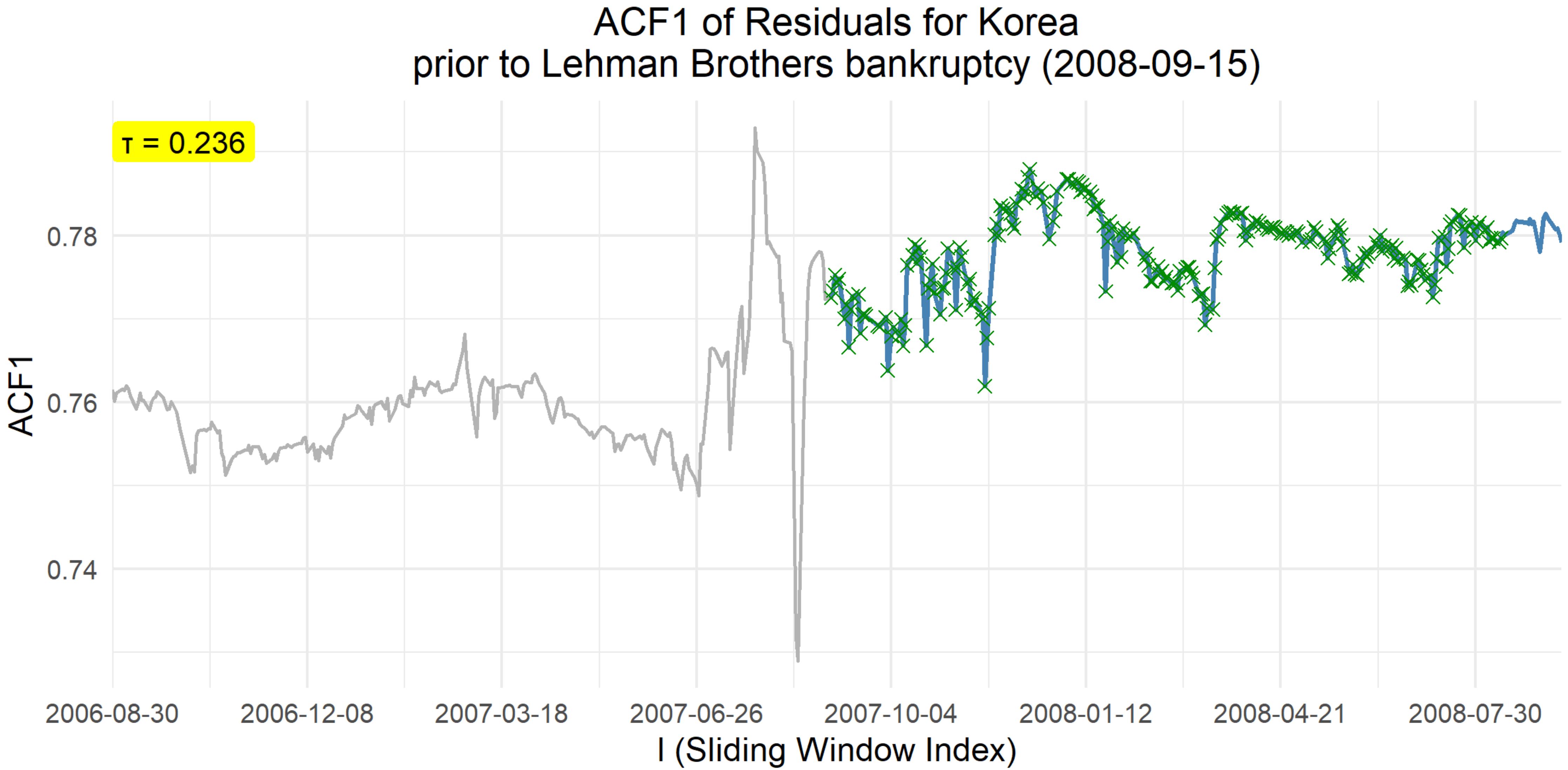
VAR on L1-norm - Korean Market



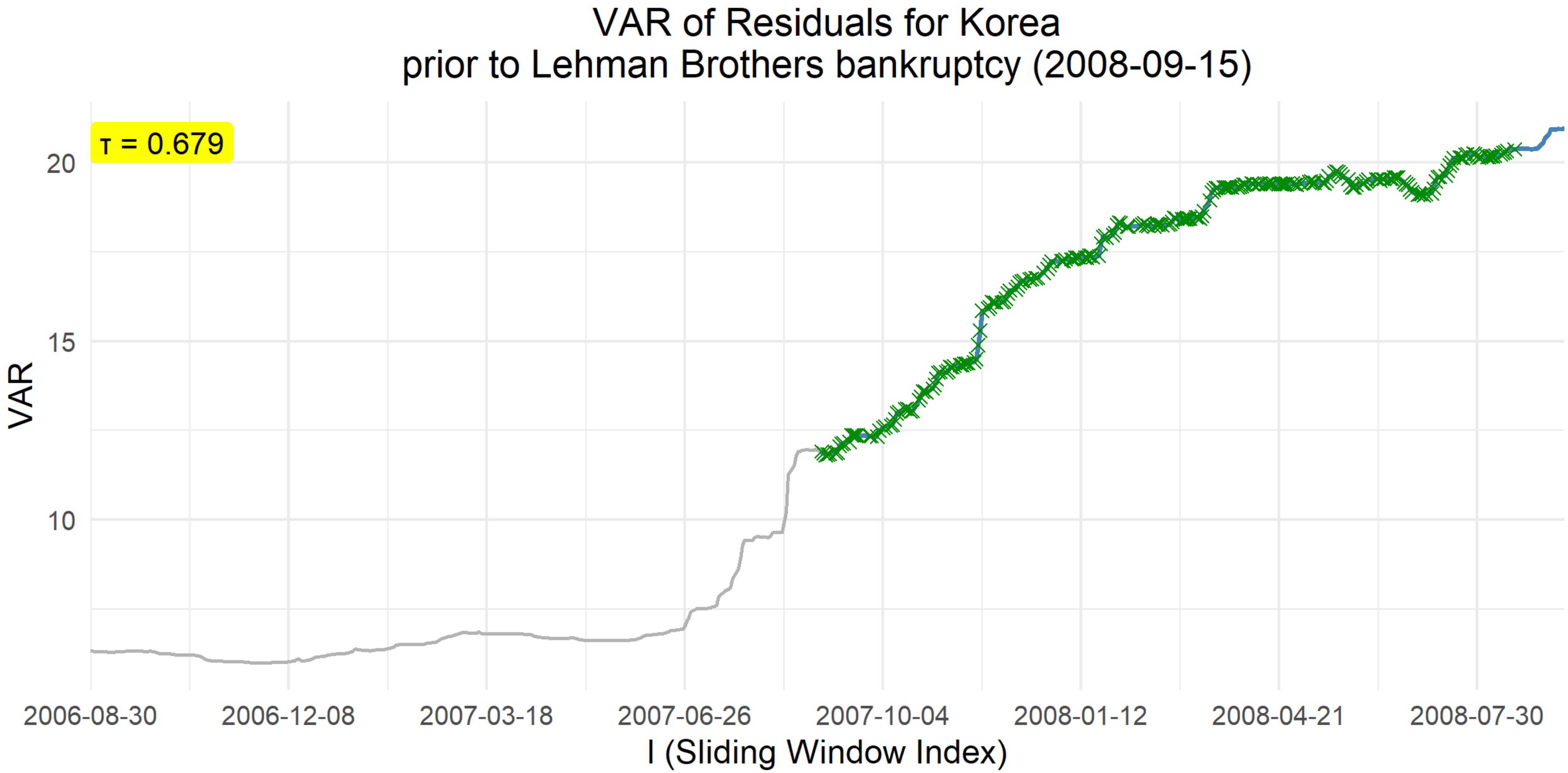
MPS on L1-norm - Korean Market



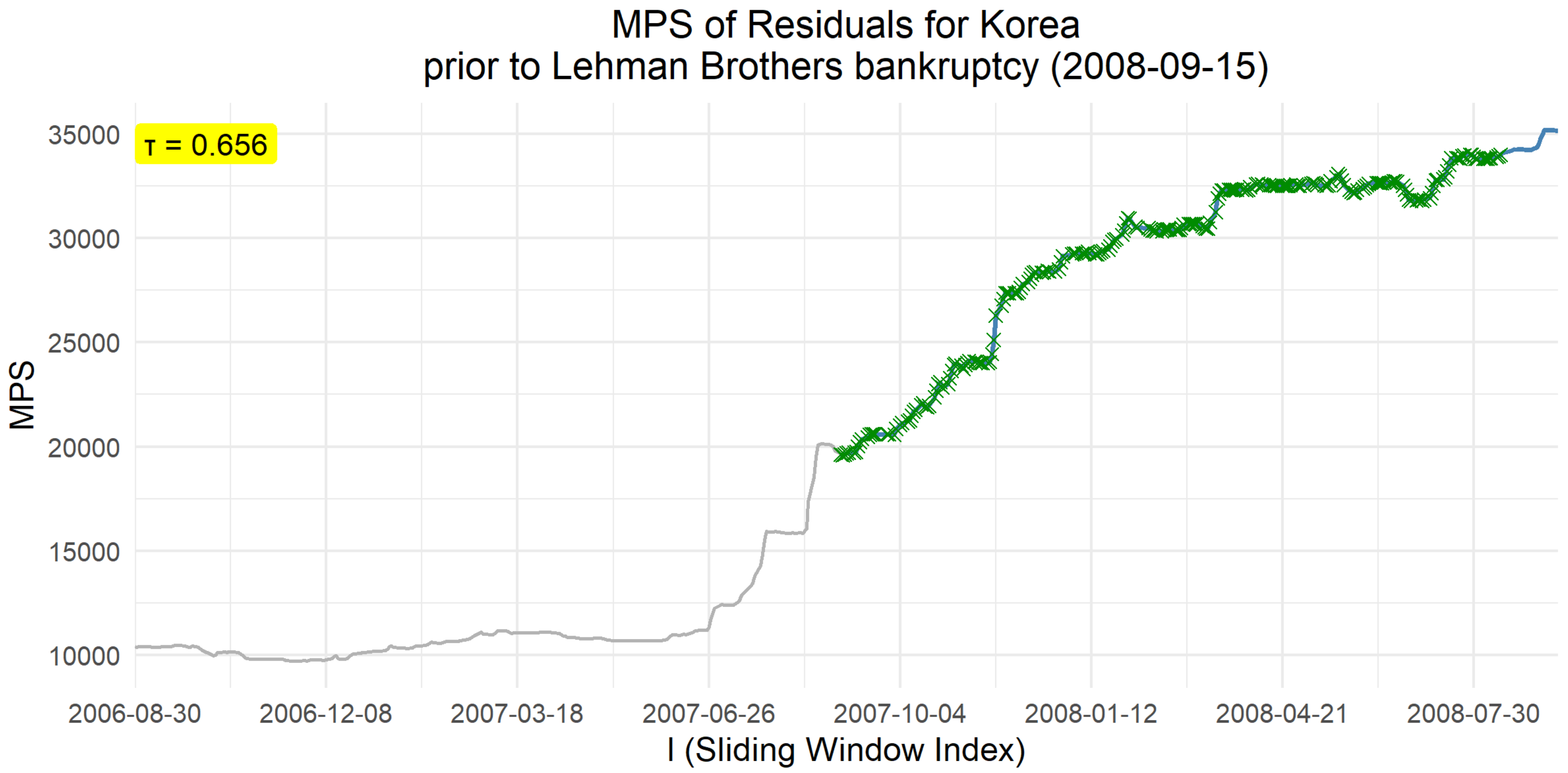
ACF1 on Residual – Korea Market



VAR on Residual – Korea Market



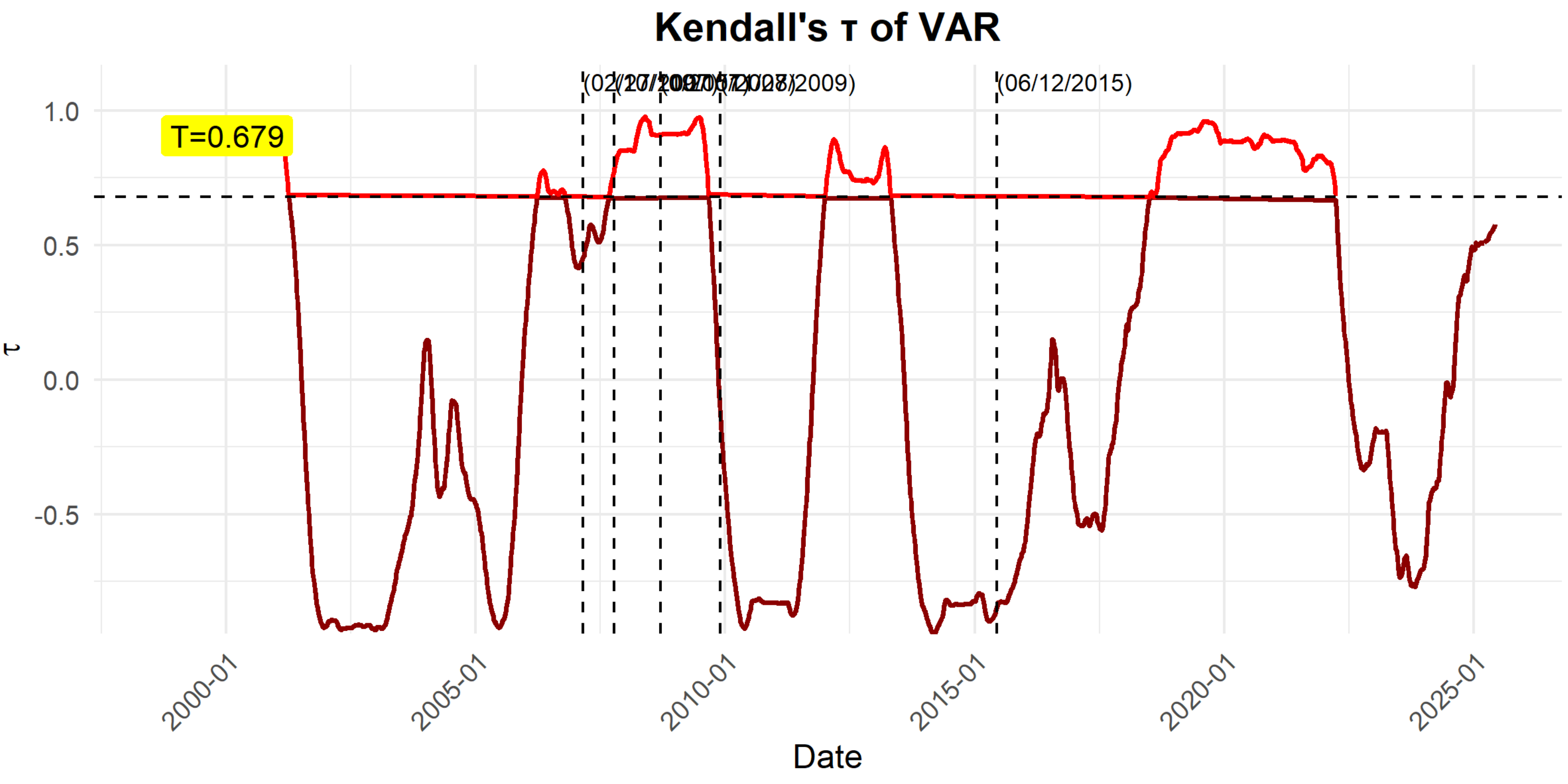
MPS on Residual – Korea Market



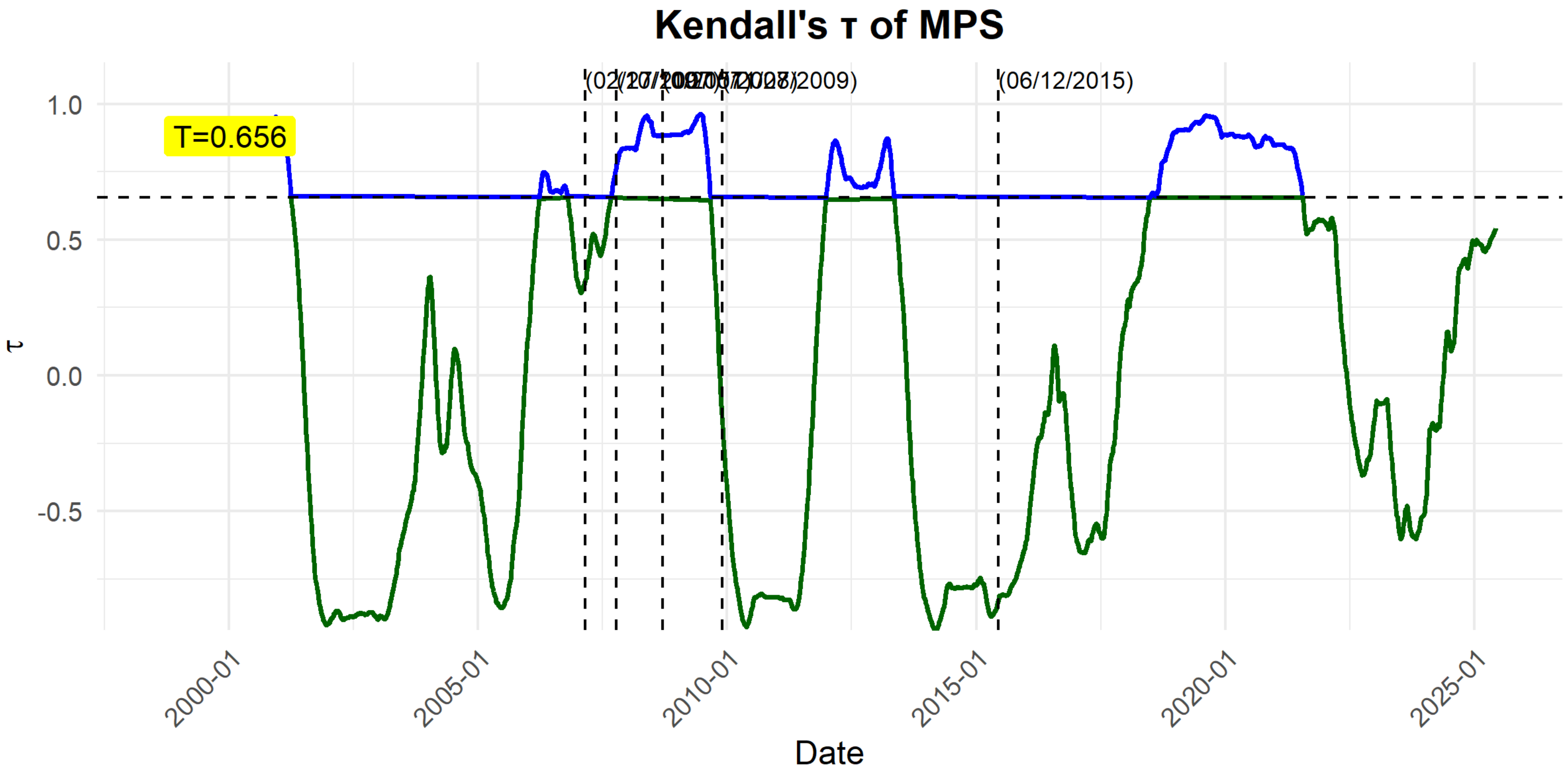
Financial Crisis

Korea
1. Chinese stock bubble of 2007 (02/27/2007) 2. US bear market of 2007 (10/11/2007) 3. Lehman Brothers Bankruptcy (09/15/2008) 4. 2009 Dubai debt standstill (11/27/2009) 5. 2015-2016 Chinese stock market crash (06/12/2015)

VAR on Residual – Korea Market



MPS on Residual – Korea Market



Results for Korea

Table 12
Summary for the classification scores obtained of the L^1-norms and the residuals.

Market	Score name	Method		
		MPS of the $\boldsymbol{L^1}$-norms	VAR of theresiduals	MPS of the residuals
US	Probability of successful anticipation (%)	60	55	**64**
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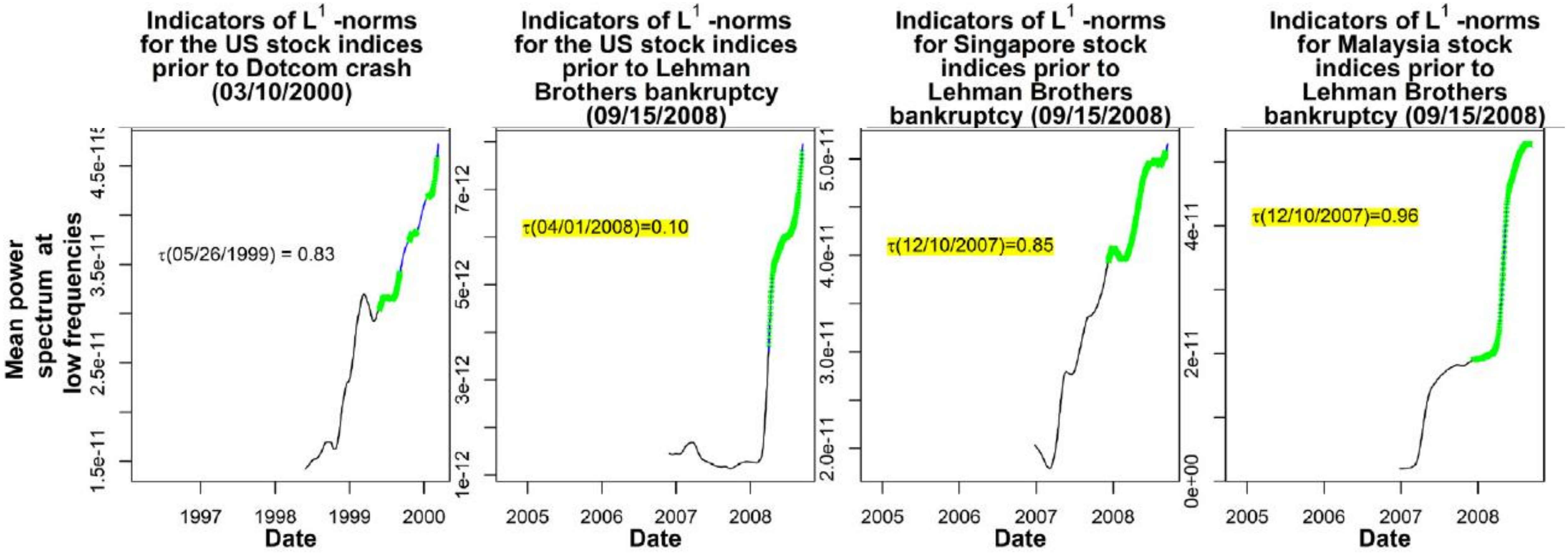
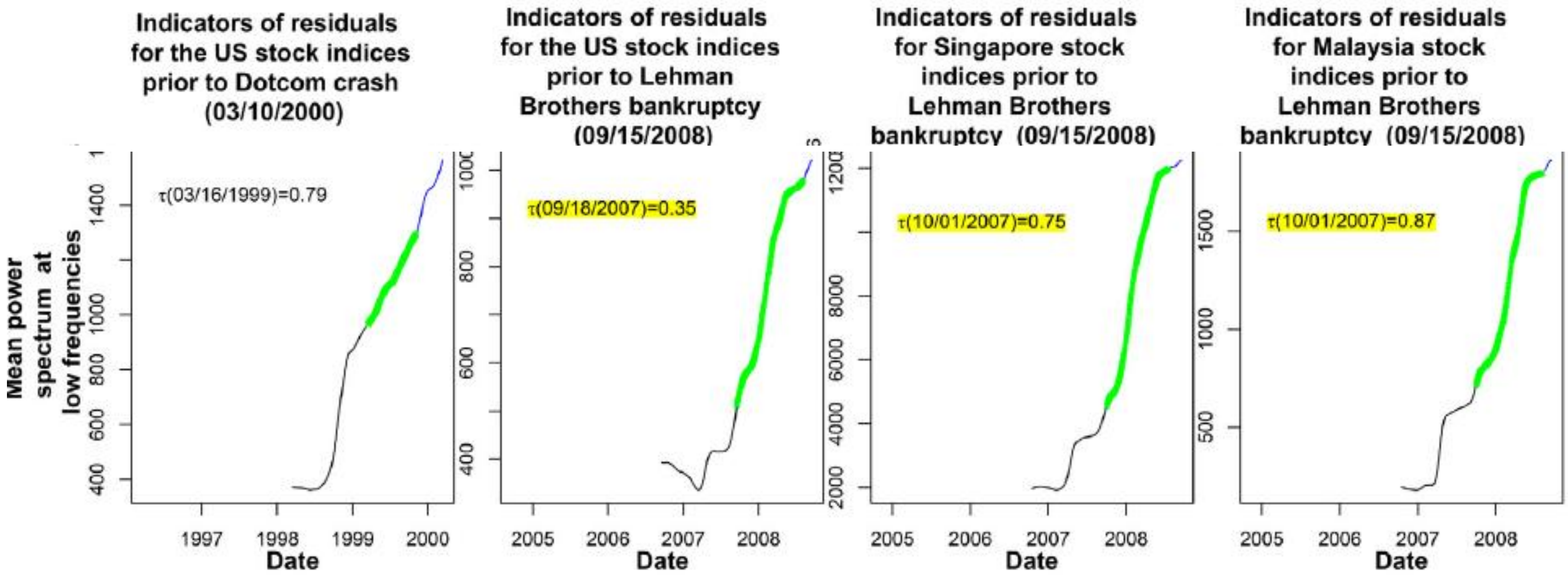
Malaysia	Probability of successful anticipation (%)	40	50	**67**
	Probability of erroneous anticipation (%)	60	50	**33**
Korea	Probability of successful anticipation (%)		**54**	**54**
	Probability of erroneous anticipation (%)		**46**	**46**

Discussions

MPS on L1-norms



MPS on Residuals



Step 7-3: Mean power spectrum (MPS) at low frequencies

- **Discrete Fourier Transform:**

$$\mathrm{F_k}(l) = \sum_{\mathrm{n=l-500+1}}^{\mathrm{l}} \mathrm{y(n)}e^{\frac{-2\pi\mathrm{ikn}}{500}}$$

where $k = 1, \ldots, 500$.

- **Power Spectrum:**

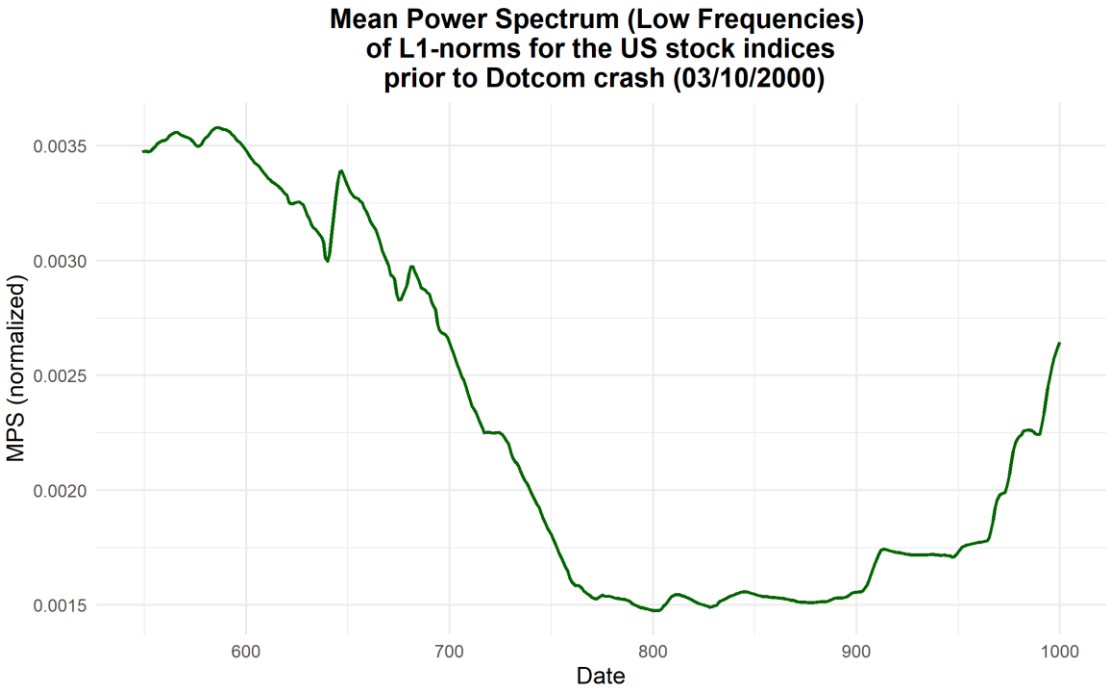
$$PS_{\mathrm{k}}(l) = |\mathrm{F_k}(l)|^2$$

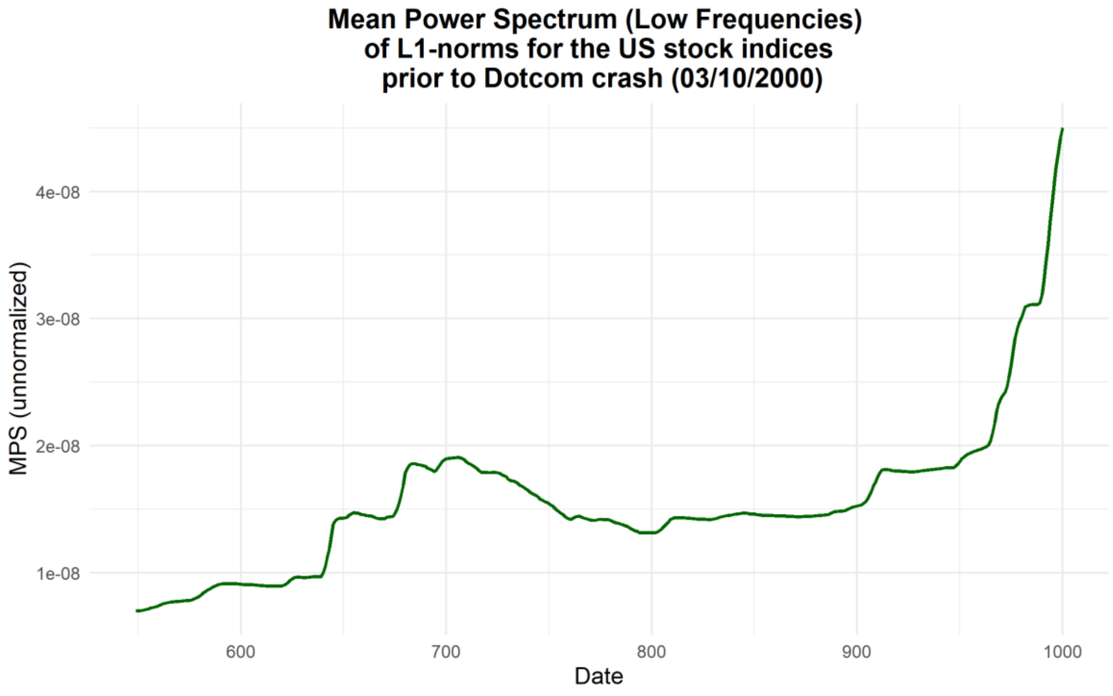
Each $PS_{\mathrm{k}}(l)$ is **normalized** such that its sum is equal to 1.

- **MPS Value:**

$$mps(l) = \frac{1}{\lfloor 500/8 \rfloor - 1} \sum_{\mathrm{k=2}}^{\lfloor 500/8 \rfloor} \mathrm{PS_k(l)}$$

MPS on L1-norms for US





Conclusions

- Persistent Homology + CSD Indicators:
  - Vietoris-Rips complex → Persistence landscape → ACF1, VAR, MPS
- PH-Based L1-Norm Time Series:
  - Strong upward trend before Dotcom & Lehman crises
- Top Methods Identified:
  - MPS on residuals (best overall)
  - MPS on L1-norms (close second)
  - VAR on residuals

Thank you